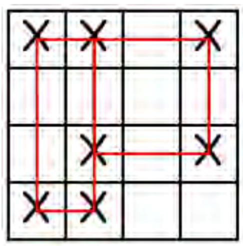
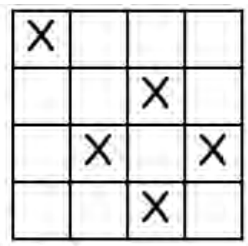
Puzzle of the Week

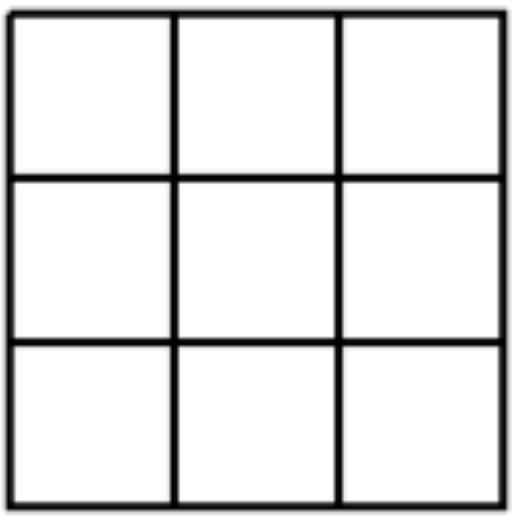
Avoiding Rectangles – 1

The X's in a grid can become the corners of rectangles with horizontal and vertical sides. The X's in this first grid form two rectangles. However, the goal is to avoid creating rectangles. The X's in the second grid are placed to avoid forming any rectangles.





THE CHALLENGE: Place as many X's as you can in this 3 by 3 grid and avoid creating any rectangles.



EXPLORATION: Look at other rectangular grids with 1, 2, or 3 rows. Do you see any patterns in your answers?

Puzzle of the Week

Avoiding Rectangles – 1 – Notes

THE CHALLENGE & EXPLORATION: Suppose the grid has m rows and n columns. The easiest first approach is to put n X's in the top row and m X's in the leftmost column. This will give you (m + n - 1) X's, and it does not produce any rectangles.

This approach does give the best answer when there are 1 or 2 rows (or columns).

Here is the best answer for a 3 by 3 grid. It has 6 X's. In terms of m and n, this has m + n X's.

X		X
X	X	
	X	X

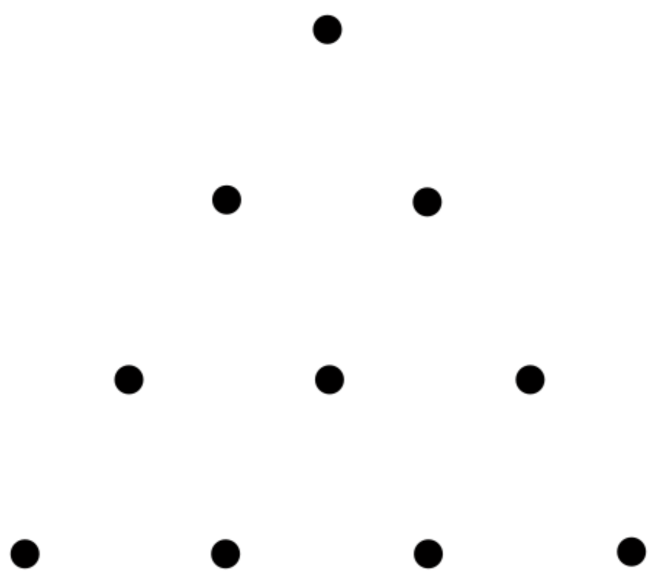
If we want to extend this to 3 by n grids, we can simply put a single X in each new column. For grids with 3 rows, we will have 3 + n X's.

We will explore this further in Avoiding Rectangles - 2.

Puzzle of the Week
Avoiding Triangles

A *regular triangle* is a triangle with equal sides and equal angles. In this first pyramid, two regular triangles have been made using the dots. In the second pyramid, enough dots have been crossed out that it is impossible to find three dots that form a regular triangle.

THE CHALLENGE: What is the fewest number of dots you can remove from this pyramid so that there are no equilateral triangles of any size or orientation formed by the dots?



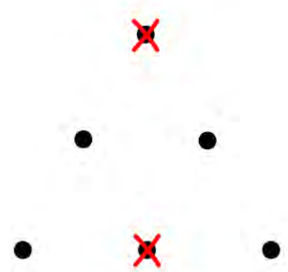
EXPLORATION: Can you cross out fewer dots than the introductory example to solve the six-dot pyramid? Do you see a pattern of solutions that would suggest solutions for larger pyramids?

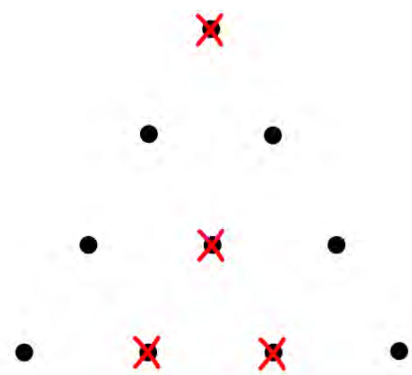
Puzzle of the Week

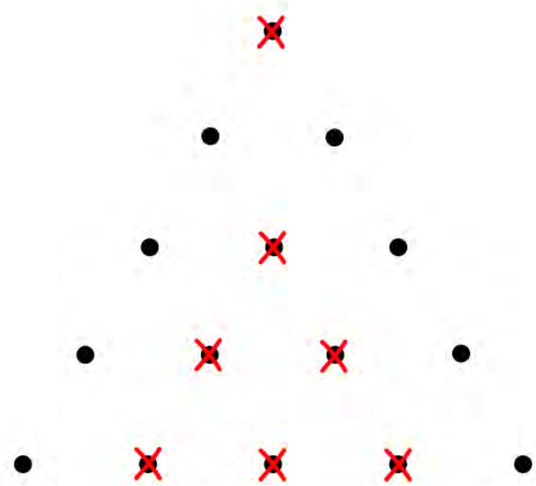
Avoiding Triangles – Notes

THE CHALLENGE: I feel comfortable that the solutions for 6-dot and 10-dot pyramids offered below are as good as possible. The designs offered are simple to understand and seem to produce a pattern of solutions.

For pyramids with 15 or more dots, it is not at all obvious that this pattern will continue to be the best possible way of doing it. Please let us know if you find any improvements.







Puzzle of the Week

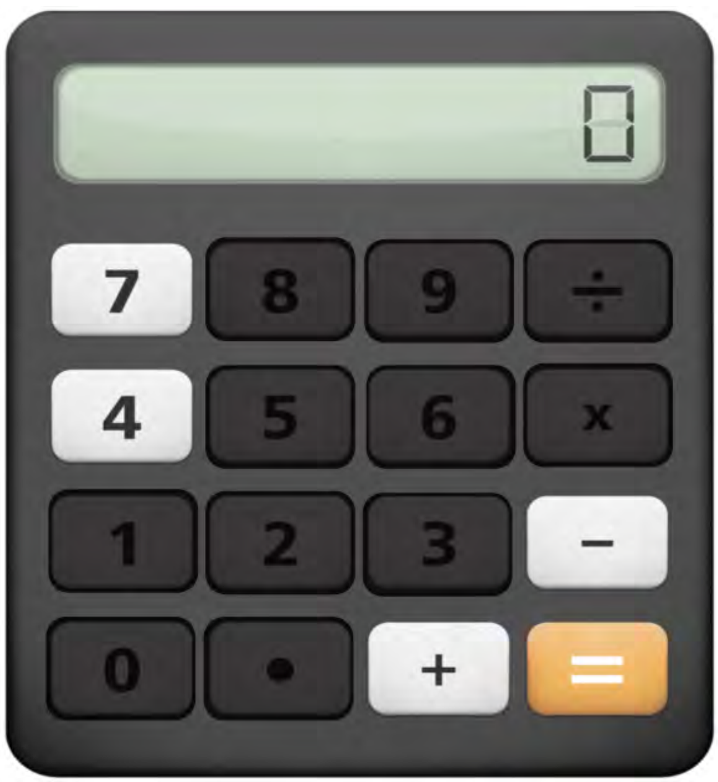
Broken Calculator – 1

You have a calculator that is badly broken. Its only working keys are 4, 7, +, and -. Even with this limited ability, it is possible to make every number. For example:

$$1 = 4 + 4 - 7$$

$$2 = 4 + 4 + 4 + 4 - 7 - 7$$

THE CHALLENGE: Show that this calculator can make all the numbers from 1 to 12.



EXPLORATION: Replace 4 and 7 by other pairs of numbers. When is it possible to get all the numbers from 1 to 12, and when is it impossible? Can you look at a pair of numbers and predict what will happen?

Puzzle of the Week

Broken Calculator – 1 – Notes

THE CHALLENGE: The students are experimenting with all the numbers that can be produced by taking any multiple of 4 and adding it to any multiple of 7. Without being told this, they are looking at what is called Bezout's Theorem. That theorem says that all possible combinations of the multiples (both positive and negative) of two numbers is the set of multiples of the greatest common factor (divisor) of the two numbers. The greatest common factor of 4 and 7 is 1, so we should be able to get all possible numbers. Furthermore, there will be an infinite number of ways to get each one.

Here are a few samples for the numbers from 1 to 12:

- 1 = 4 + 4 - 7 = 9 x 4 - 5 x 7 = 16 x 4 - 9 x 7
- 2 = 7 + 7 - 4 - 4 - 4 = 6 x 7 – 10 x 4 = 10 x 7 - 17 x 4
- 3 = 7 - 4 = 5 x 7 - 8 x 4 = 9 x 7 - 15 x 4
- 4 = 4 = 4 x 7 - 6 x 4 = 8 x 7 - 13 x 4
- 5 = 3 x 4 - 7
- 6 = 2 x 7 - 2 x 4
- 7 = 7
- 8 = 2 x 4
- 9 = 3 x 7 - 3 x 4
- 10 = 2 x 7 - 4
- 11 = 4 + 7
- 12 = 3 x 4

Once you have a combination for the greatest common factor, you can use it to get any multiple of the greatest common factor. For 4 and 7, one way to get 1 is as 2 x 4 - 7. We can get any other number, such as 23, by writing 23 = 23 (2 x 4 - 7) = 46 x 4 - 23 x 7. Once we have one solution for a number, we can get all the other solutions by adding 0 to it (I know that sounds odd). For example, we have 1 = 2 x 4 - 7 = (2 x 4 - 7) + (7 x 4 - 4 x 7) = 9 x 4 - 5 x 7). We can add any multiple we like of (7 x 4 - 4 x 7) and it won't change the value.

EXPLORATION: As mentioned above, this process will produce exactly all the multiples of the greatest common factor of the two numbers. If that GCF is 1, then all numbers will be produced.

Take 4 and 6 for example. Their GCF is 2. Any combination of multiples of 4 and 6 will always be even, so it is impossible to produce every number. This becomes more apparent if you think of 4 and 6 as 2 times 2 and 3. You know that sums of multiples of 2 and 3 will produce all numbers because they are relatively prime. However, 4 and 6 will take each of those sums and multiply them by 2, so that's why you will get all multiples of 2!

Puzzle of the Week

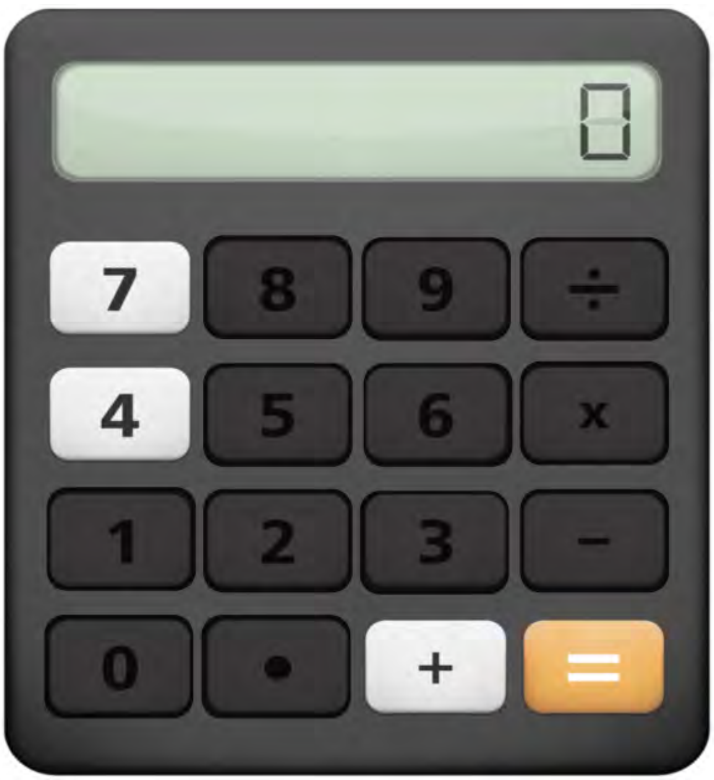
Broken Calculator – 2

You have a calculator that is badly broken. Its only working keys are 4, 7, and +. With this limited ability, it is still possible to make certain numbers. For example:

$$15 = 7 + 4 + 4$$

$$25 = 7 + 7 + 7 + 4.$$

THE CHALLENGE: This calculator cannot make 1, 2, 3, 5, and 6. Find the largest number that cannot be made by this calculator.



EXPLORATION: Replace 4 and 7 by other pairs of numbers. Describe some patterns of what you see.

Puzzle of the Week

Broken Calculator – 2 – Notes

THE CHALLENGE & EXPLORATION: This is trickier than what happens in "Broken Calculator - 1." The math theorem involved is affectionately called the McNugget Theorem. Suppose you have two relatively prime numbers n and m - that is, their greatest common factor (divisor) is 1. The theorem says that you can produce exactly half of the numbers from 1 to (n - 1) x (m - 1) as sums of positive multiples of n and m, and that starting with (n - 1) x (m - 1) you can produce every number thereafter.

Applying this theorem to 4 and 7, we know that we can produce half the numbers up to (4 - 1) x (7 - 1) = 18, and that from then on we can make all numbers. Let's see how that works in this special case.

- 1 = Impossible
- 2 = impossible
- 3 = impossible
- 4 = 4
- 5 = impossible
- 6 = impossible
- 7 = 7
- 8 = 4 + 4
- 9 = impossible
- 10 = impossible
- 11 = 4 + 7
- 12 = 4 + 4 + 4
- 13 = impossible
- 14 = 7 + 7
- 15 = 4 + 4 + 7
- 16 = 4 + 4 + 4 + 4
- 17 = impossible
- 18 = 4 + 7 + 7

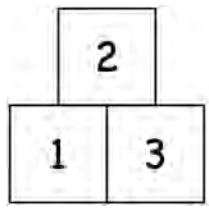
Without understanding any fancy number theory, the intrepid explorer might notice that half the numbers from 1 to 18 are possible and the other half are impossible. They might also notice the pattern that x is possible exactly when 17 - x is possible.

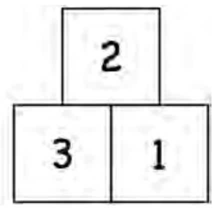
Another pattern to notice is that any common multiple of the two numbers will divide all of the sums of the multiples of the two numbers. This means that if the two numbers have a common multiple greater than 1, then there will be an infinite number of numbers that cannot be produced by that calculator. For example, combinations of 4 and 6 will never produce an odd number.

Puzzle of the Week

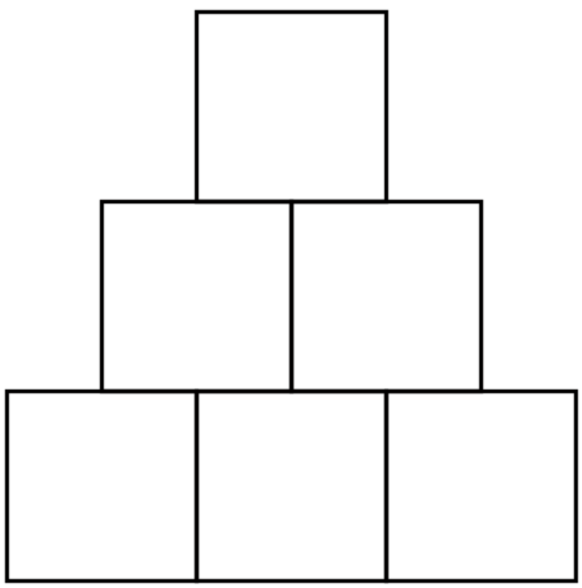
Difference Pyramids – 1

These pyramids are called *Difference Pyramids*. The number on top is the difference of the two numbers below.





THE CHALLENGE: Place the numbers from 1 to 6 to make a Difference Pyramid.



1 2 3 4 5 6

EXPLORATION: Find different ways this can be done. Are some of these essentially the same?

Puzzle of the Week

Difference Pyramids – 1 – Notes

THE CHALLENGE & EXPLORATION: A good start is to realize that because 6 cannot be the difference of two cards, it must go on the bottom row.

Next, the only way 5 can be the difference is if it is above the 6 and the 1. So, either 5 goes directly above the 6 (with the 1 next to the 6), or 5 is in the bottom row.

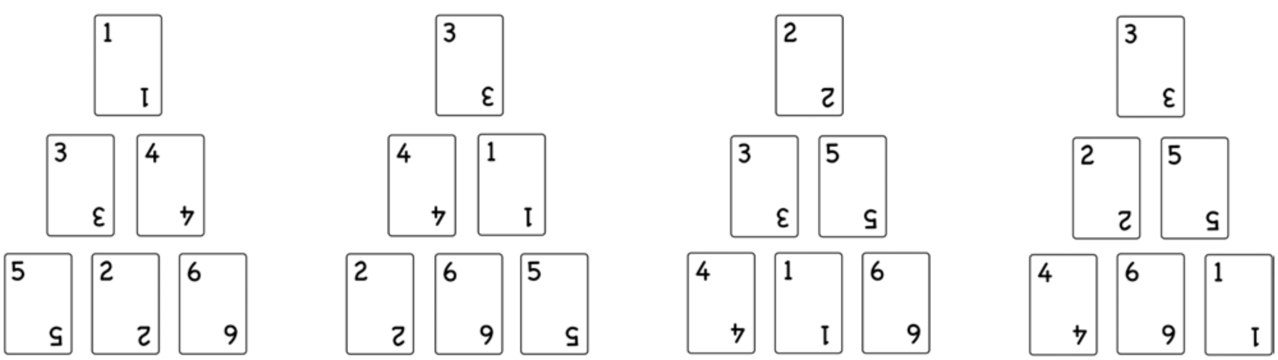
At this point it is useful to consider what makes solutions different. Because the mirror image of any solution is also a solution, it makes sense to ignore those. Ignoring mirror images will reduce the number of solutions to consider by half.

For example, we can assume that not only is the 6 in the bottom row, but it is either in the middle or the right side of the bottom row - if it were on the left side, we could take the mirror image of the whole puzzle and put it on the right side.

Using this thinking, the bottom row can have five possible layouts (up to using mirror images): 5 X 6, X 5 6, X 6 5, X 1 6, X 6 1.

At this point, it is a matter of working through the various possibilities. The only 5 X 6 that works is 5 2 6. It turns out X 5 6 never works. The only X 6 5 is 2 6 5. The only X 1 6 is 4 1 6, and the only X 6 1 is 4 6 1.

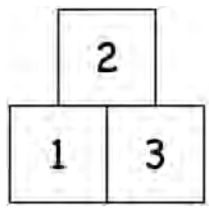
So, ignoring mirror images, there are four solutions:

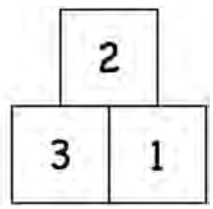


Puzzle of the Week

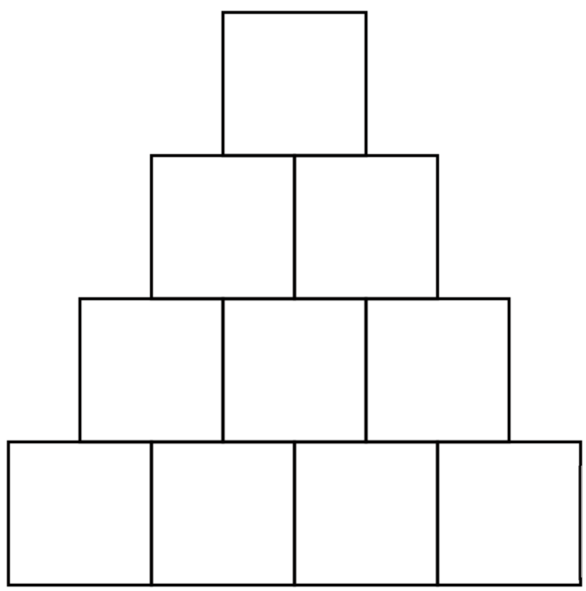
Difference Pyramids – 2

These pyramids are called *Difference Pyramids*. The number on top is the difference of the two numbers below.





THE CHALLENGE: Place the numbers from 1 to 10 to make a Difference Pyramid.



1 2 3 4 5 6 7 8 9 10

EXPLORATION: Play with even larger pyramids.

Puzzle of the Week

Difference Pyramids – 2 – Notes

THE CHALLENGE: Because 10 cannot be the difference of two cards, it must go on the bottom row. Similarly, either 9 is in the bottom row or it is in the next-to-the-bottom row above the 1 and the 10. The 8 and 7 cards are also good cards to focus on to get rid of possibilities.

This means the bottom row looks like one of the following (ignoring mirror images):

X Y 9 10,   X 9 Y 10,   9 X Y 10,   X Y 10 9,   X 9 10 Y,   9 X 10 Y,   X Y 1 10,   X 1 10 Y, X Y 10 1

That is a lot of possibilities to consider!

Fortunately, if you consider where 8 and 7 can go, the possibilities are reduced to the following list (assuming we haven't missed any). It is easy to finish each of these once you have the bottom row.

8 3 10 9,   9 3 10 8,   6 1 10 8,   8 1 10 6

Investigating pyramids of size 15, 21, or higher are for the truly dedicated.

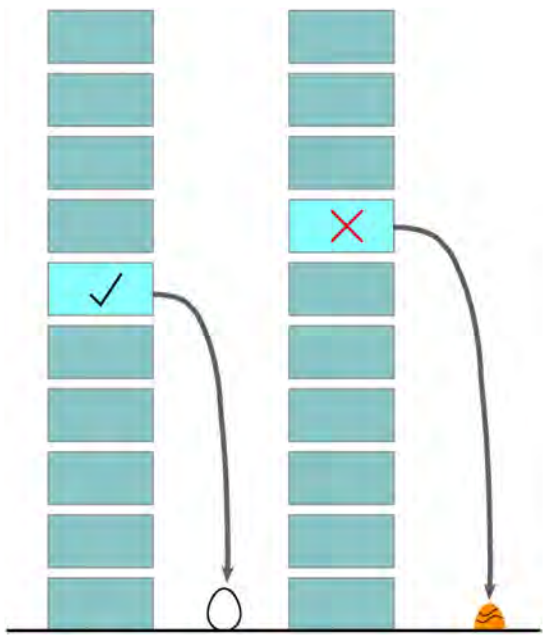
From the literature, there is only one solution (up to reflections) for 15, and its bottom row is {6, 14, 15, 3, 13}. Even more surprisingly, there are no solutions for 21, 28, and 36, and it has been conjectured that there are no solutions above that!

Puzzle of the Week

Egg Drop to 10

There is a new type of single-egg container to test. You are given a container, two eggs, and access to a ten-story building. You want to see how high up you can drop an egg in one of these containers and not have the egg break.

You could simply drop an egg from each floor, starting with the first floor, and see where the egg breaks for the first time. However, if the containers are really good, that would mean ten egg drops. How can you find out how good the container is with fewer drops?



THE CHALLENGE: Find a method that uses the fewest total number of drops, no matter how good or bad this new container is, to discover the highest floor in this ten-story building you can safely drop an egg from.

EXPLORATION: How would your strategy change if you had three eggs to work with?

Puzzle of the Week

Egg Drop to 10 – Notes

THE CHALLENGE & EXPLORATION: The first possible solution is to split the problem in half and drop the first egg from the fifth floor. In the worst case, when the container works all the way up to the tenth floor, there would only be six drops.

An improvement to that solution is to split it into multiple first steps - say the 3rd and 6th floor. However, that still creates a worst case of 6 drops. Using 4th and 8th floors improves it to five drops.

Five drops is as good as it gets with two eggs.

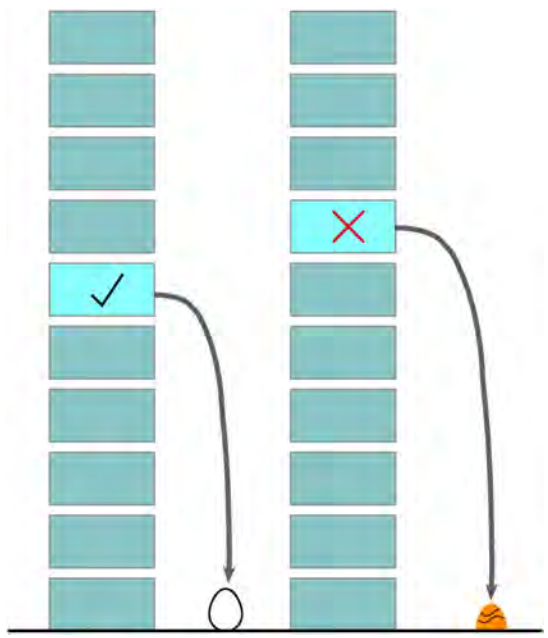
With three eggs, we could first do drops at the 4th and 8th floors. Then, as appropriate, next drops at 2nd and 6th floors. That would reduce the total number of drops to four drops in the worst case.

Puzzle of the Week

Egg Drop to 100

There is a new type of single-egg container to test. You are given a container, two eggs, and access to a one hundred-story building. You want to see how high up you can drop an egg in one of these containers and not have the egg break.

You could simply drop an egg from each floor, starting with the first floor, and see where the egg breaks for the first time. However, if the containers are really good, that would mean one hundred egg drops. How can you find out how good the container is with fewer drops?



THE CHALLENGE: Find a method that uses the fewest total number of drops, no matter how good or bad this new container is, to discover the highest floor in this one hundred-story building you can safely drop an egg from.

EXPLORATION: How would your strategy change if you had three eggs to work with?

Puzzle of the Week

Egg Drop to 100 – Notes

THE CHALLENGE: In "Egg Drop – 1" we saw that using evenly spaced floors for the first round of tests reduced the total number of drops.

For example, if the first round involved dropping from multiples of ten, then the worst case would require 19 drops – 9 up to the 90th floor, and then single floor skips up to the 100th floor.

The big improvement comes from deciding to use variable-sized steps during the first phase. If you space them evenly, then by the time you have gotten near the top, you have all the tests to get up there and then a bunch of single-step tests after that. The key is to reduce the step size by one after each test during the first phase.

While you can figure out the first step size mathematically, it is easy enough to do by trial and error. If you start with a step size of 12 and decrease it by 1 each time, you will have steps of size 12 + 11 + 10 + …. + 1 = 78, and you will not get to the top. You need to start with a step size of 14 to make it. Now, the worst case is 14 drops, which is a big improvement!

EXPLORATION: Using three eggs gets tricky. Each time you add a new egg, you want to take advantage of knowing the best way to use the earlier collection of eggs. For example, when using two eggs, you know that after the first egg fails you will be testing in single steps.

Doing 3 eggs rigorously takes an unreasonable amount of math for an elementary school student. However, we can substitute some trial and error and still get the answer reasonably quickly.

Let's start by knowing these sums (called triangular numbers): 1 + 2 = 3, 1 + 2 + 3 = 6, 1 + 2 + 3 + 4 = 10, 1 + 2 + 3 + 4 + 5 = 15, 1 + 2 + 3 + 4 + 5 + 6 = 21, 1 + 2 + 3 + 4 + 5 + 6 + 7 = 28, and 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36. We want to use these sums to guide our choices of initial tests. For example, if we start with28, and then use 28 + 21 = 49, and then go to 28 + 21 + 15 = 64, and so on, we won't quite get to the top.

So, our first set of tests should be at 36, 36 + 28 = 64, 36 + 28 + 21 = 85, 36 + 28 + 21 + 15 = 100. That is, we will test at floors 36, 64, and 85. No matter how those tests go, the worst case scenario is that we will need 9 tests!

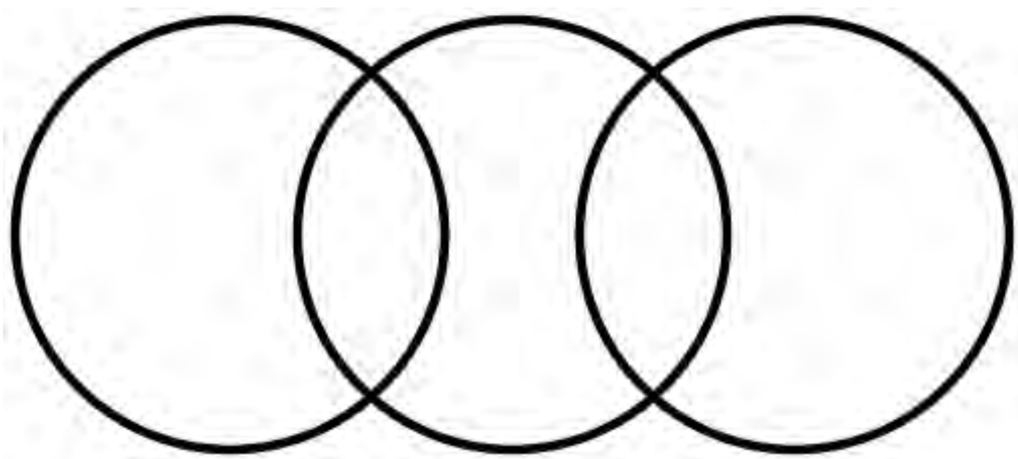
You may also enjoy showing your class the following video from TEDED:

https://ed.ted.com/lessons/can-you-solve-the-egg-drop-riddle-yossi-elran

Puzzle of the Week

Equal Sums – 1

THE CHALLENGE: Here is a diagram created by overlapping three circles. The overlapping circles create five regions. Put a number in each of the five regions, using each of the numbers 1 to 5 exactly once, so that the sum of the numbers in each circle is the same.



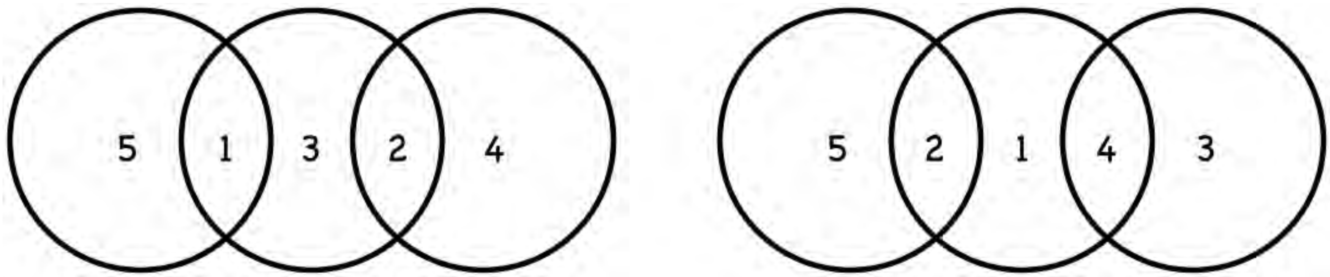
1 2 3 4 5

EXPLORATION: How many different answers can you find? How do you know if you have found them all?

Puzzle of the Week

Equal Sums – 1 – Notes

THE CHALLENGE & EXPLORATION: There are two solutions. Going from left to right, the sum in each circle is 6 and 7.



Analyze the possibilities by letting A and B be the two numbers in the intersections of the circles. Let Sum be the common sum inside each circle. Then 3 x Sum = 1 + 2 + 3 + 4 + 5 + A + B = 15 + A + B.

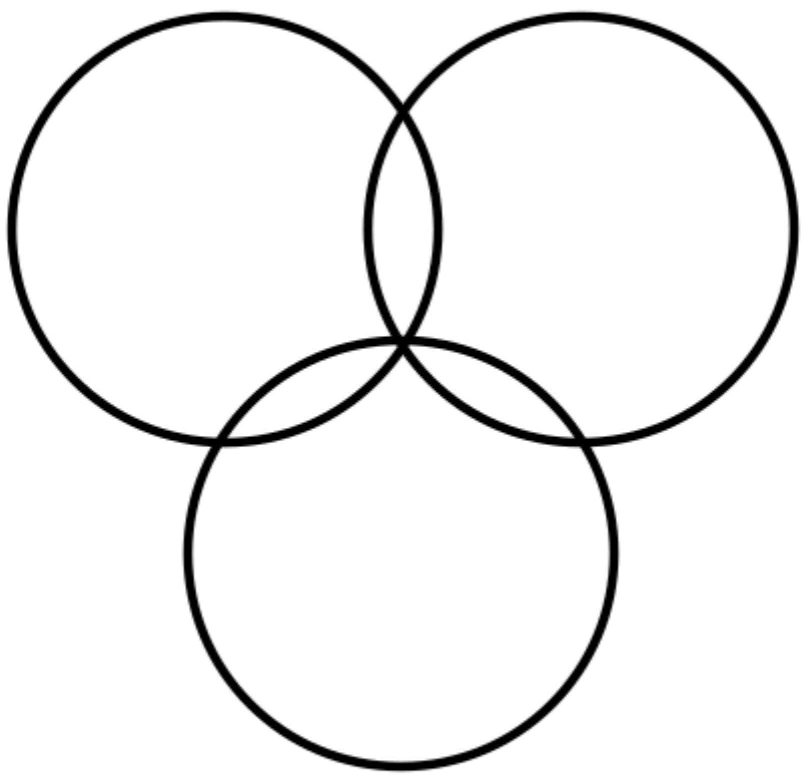
The left side of 3 x Sum = 15 + A + B is a multiple of 3, so the right side is as well. This forces A + B to be a multiple of three. That leaves only three possibilities.

- A + B = 3. In this case 3 x Sum = 15 + 3 = 18 tells us Sum = 6, and A and B are 1 and 2.
- A + B = 6. In this case 3 x Sum = 15 + 6 = 21 tells us Sum = 7. A + B = 6 forces A and B to be either 1 and 5 or 2 and 4. Having A and B be 1 and 5 does not work (1 is repeated), so that leaves us with just 2 and 4.
- A + B = 9. In this case 3 x Sum = 15 + 9 = 24 tells us Sum = 8, and A and B are 4 and 5. However, because A and B are both in the middle circle, it is not possible for A + B = 9 and yet the Sum is only 8. So this case cannot happen.

Puzzle of the Week

Equal Sums – 2

THE CHALLENGE: Here is a diagram created by overlapping three circles. The overlapping circles create six regions. Put a number in each of the six regions, using each of the numbers 1 to 6 exactly once, so that the sum of the numbers in each circle is the same.



1 2 3 4 5 6

EXPLORATION: How many different answers can you find? How do you know you have found them all?

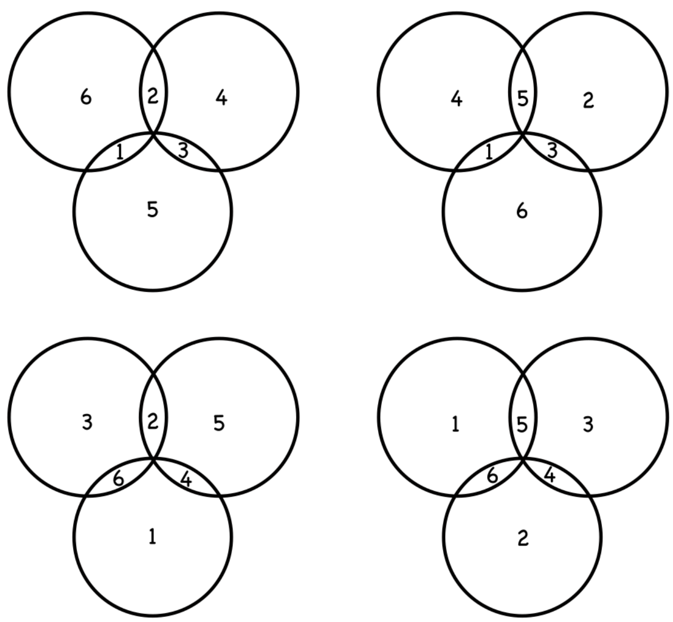
Puzzle of the Week

Equal Sums – 2 – Notes

THE CHALLENGE & EXPLORATION: Here are the four solutions. To see why these are the only ones, let A, B, and C be the three regions where two circles intersect, and let Sum be the common sum for the three circles. Calculate the total sum in two ways. As the sum of the three circles, the sum is 3 x Sum. As the sum of all six numbers plus A, B, and C the sum is 1 + 2 + 3 + 4 + 5 + 6 + A + B + C. The two things are equal, so that gives us 3 x Sum = 21 + A + B + C. Dividing by 3, we have Sum = 7 + (A + B + C)/3.

For Sum to be an integer in this last equation, A + B + C must be evenly divisible by 3. This allows for only four possibilities for A + B + C - it is either 6, 9, 12, or 15, and the corresponding values for Sum are 9, 10, 11, or 12.

The four solutions given here have Sum values of 9, 10, 11, and 12. As we shall see, these are all of the solutions!



To save some work, note that if we take one solution and subtract all the entries from 7, we get another solution. Also, note that the new solution will have a Sum value which is 21 - (old Sum). Hence, the solutions for Sum values 9 and 10 give us the solutions for Sum values 11 and 12.

If A + B + C = 6, then A, B, and C are 1, 2, and 3, and Sum = 9. That is the upper left solution.

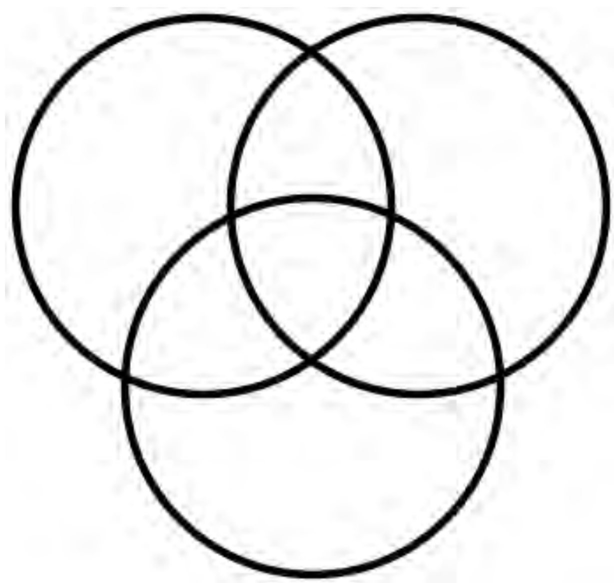
If A + B + C = 9, then Sum = 10 and (A, B, C) is (1, 2, 6), (1, 3, 5), or (2, 3, 4). If you check it, (1, 2, 6) and (2, 3, 4) are impossible. This leaves the upper right solution as the only Sum = 10 solution.

If you compare “Equal Sums – 2” with “Magic Triangles – 1,” you will see that they are the same puzzle!

Puzzle of the Week

Equal Sums – 3

THE CHALLENGE: Here is a diagram created by overlapping three circles. The overlapping circles create seven regions. Put a number in each of the seven regions, using each of the numbers 1 to 7 exactly once, so that the sum of the numbers in each circle is the same.



1 2 3 4 5 6 7

EXPLORATION: How many different answers can you find? How do you know you have found them all?

Puzzle of the Week

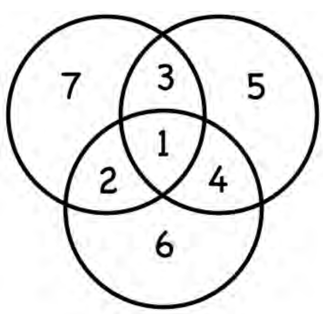
Equal Sums – 3 – Notes

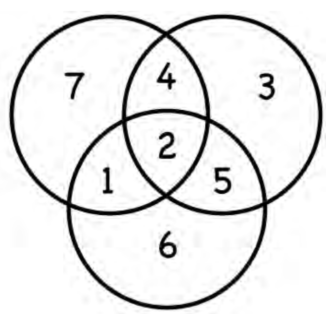
THE CHALLENGE & EXPLORATION: The first question with these problems is what the possible sums in the circles are. Narrow this down by looking at the sum of the entries in two ways. Let A, B, and C be the three values where two circles intersect, let M be the value where all three circles intersect, and let Sum be the common sum in each circle. The sum of the three circles is 3 x Sum. That sum is also the sum of the numbers from 1 to 7 plus A + B + C plus 2 x M. Therefore, 3 x Sum = 28 + A + B + C + 2 x M.

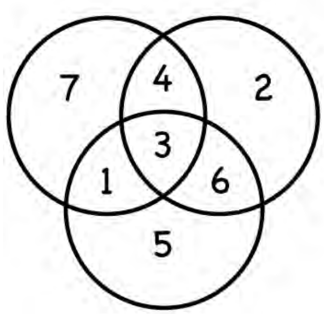
Before going further with this, note that if we start with any solution, we can get a new solution by subtracting all the entries from 8. This will have the effect of replacing Sum by 32 - Sum. Because of this, we only need to look for values of Sum up to 16 - the remaining larger values can be obtained by subtracting those solution entries from 8.

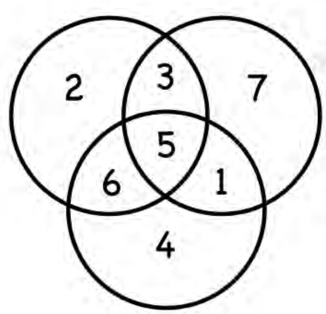
The smallest A + B + C + 2 x M can be is 2 + 3 + 4 + 2 x 1 = 11. So the smallest Sum can be is (28 + 11) / 3 = 13. Consequently, we want to see which of the Sum values from 13 to 16 are possible. It turns out that they are all possible.

Here is one solution for each Sum value starting at 13 and ending at 16. There are many more.





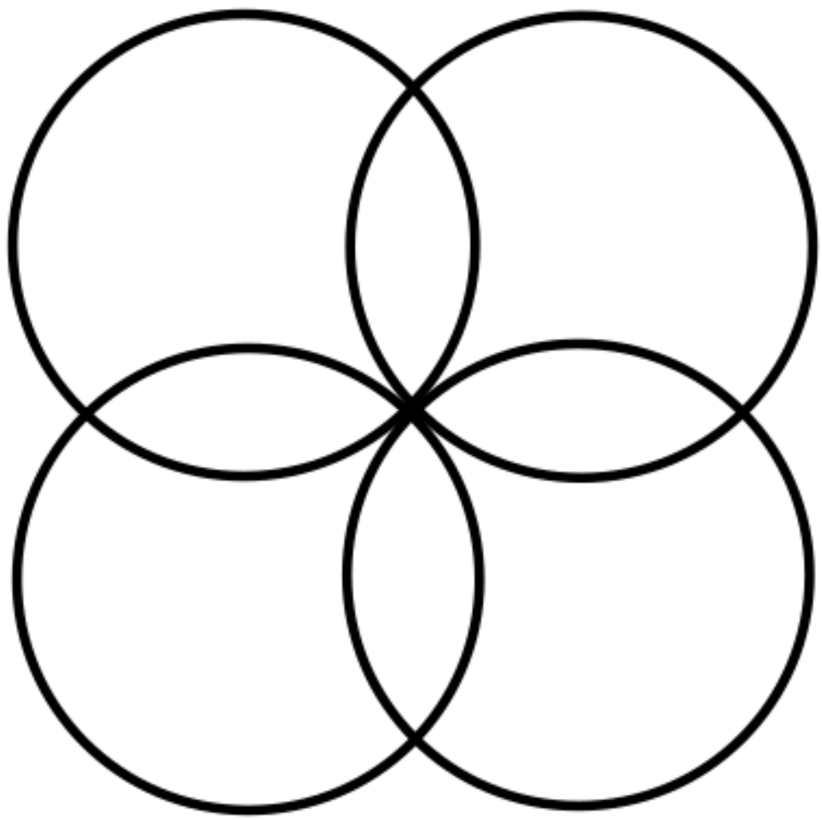




Puzzle of the Week

Equal Sums – 4

THE CHALLENGE: Here is a diagram created by overlapping four circles. The overlapping circles create eight regions. Put a number in each of the eight regions, using each of the numbers 1 to 8 exactly once, so that the sum of the numbers in each circle is the same.



1 2 3 4 5 6 7 8

EXPLORATION: How many different answers can you find? Do you think there are any more? What happens if you use other number ranges? Are there other interesting problems like this with intersecting circles?

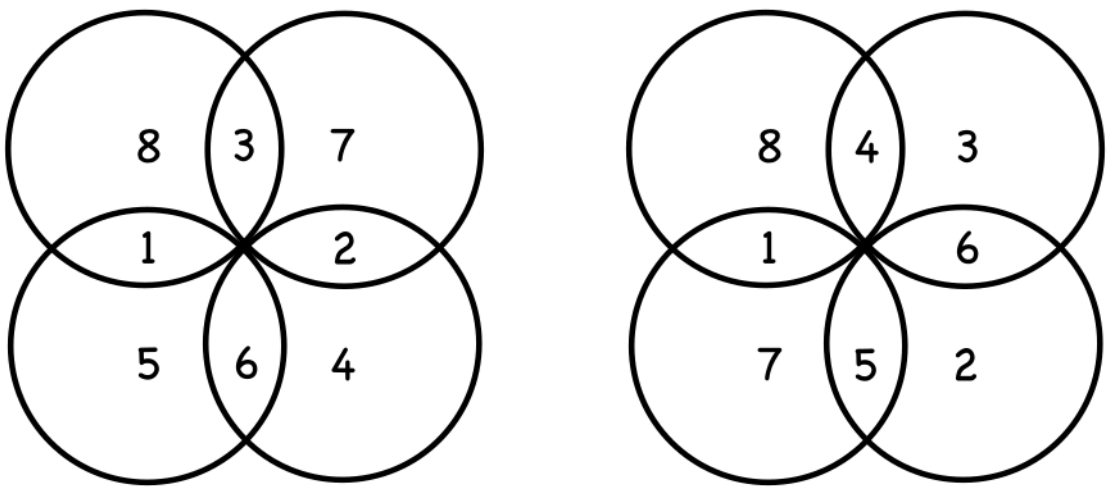
Puzzle of the Week

Equal Sums – 4 – Notes

THE CHALLENGE & EXPLORATION: Start by deciding which sums are possible for the common sum for all circles. Let Sum be that value. Let A, B, C, and D be the values of the four regions where two circles intersect. The sum of the four circles is 4 x Sum, and it is also 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + A + B + C + D. Then we have the equation 4 x Sum = 36 + A + B + C + D, so Sum = 9 + (A + B + C + D) / 4. The smallest that A + B + C + D can have is 1 + 2 + 3 + 4 = 10. This needs to be divisible, so it's smallest possible value is 12, which means the smallest possible value is Sum = 9 + 12/4 = 12.

Note that any solution can be turned into another solution by subtracting all entries from 9. Doing this will turn the old Sum into 27 - Sum. Consequently, the only possible values for Sum are 12, 13, 14, and 15. Because 12 and 15 are tied together, and 13 and 14 are tied together, we only need to check for solutions for 12 and 13.

Here are solutions for Sum with values 12 and 13. It turns out these are the only ones for these two sums.



Notice that the numbers in the center of circles not touching each other add up to the same thing. For example, in the leftmost diagram, 8 + 4 = 5 + 7. Prove that this always happens with a bit of algebra. Let K and L be the values in the centers of two of the opposite circles, and let M and N be the other two values. If we add up the regions for the two circles for K and L we get 2 x Sum = K + L + (A + B + C + D). Similarly, adding up the regions for the two circles for M and N gives 2 x Sum = M + N + (A + B + C + D). This forces K + L = M + N.

Also, note that since 4 x Sum = 36 + A + B + C + D, we get 2 x Sum = M + N + (4 x Sum - 36). Rewriting this we have M + N = 36 - 2 x Sum. For our four values of Sum, 12 through 15, the values of M + N are 12, 10, 8, and 6.

Because 12 = 4 + 8 = 5 + 7 are the only ways to get 12, the solution above for Sum = 12 is the only solution. While 10 = 2 + 8 = 3 + 7 = 4 + 6 suggests there are more possibilities for Sum = 13, a quick check of the three possible pairings of (2, 8), (3, 7), and (4, 6) shows that the solution above is the only one for Sum = 13.

Puzzle of the Week

Fill in the Blanks – 1

Using the numbers from 1 to 5 at most once, this equation has three solutions.

$$\square - \square = \square - \square$$

1 2 3 4 5

The three solutions are:

$$\boxed{3} - \boxed{1} = \boxed{4} - \boxed{2}$$

$$\boxed{4} - \boxed{2} = \boxed{5} - \boxed{3}$$

$$\boxed{4} - \boxed{1} = \boxed{5} - \boxed{2}$$

THE CHALLENGE: Use the numbers from 1 to 6 at most once to fill in these blanks.

$$\square + \square = \square - \square$$

1 2 3 4 5 6

EXPLORATION: Explore other number ranges. What happens if you use 1 to 5, 1 to 7, or 1 to 8? How do things change if you use 0 to 6?

Puzzle of the Week

Fill in the Blanks – 1 – Notes

THE CHALLENGE: As with the other Fill in the Blanks puzzles, a child can just play with this and eventually arrive at the answers. That exploration involves a lot of good experiences, and there is no reason to avoid it.

To be more systematic, the key observation is that the subtraction drives the solution.

Here are the solutions written out in order of the difference:

- 2 + 3 = 6 - 1
- 1 + 3 = 6 - 2
- 1 + 2 = 6 - 3

EXPLORATION: There are no solutions for the range 1 to 5.

For 1 to 7, there are all the solutions above plus these:

- 2 + 4 = 7 - 1
- 1 + 4 = 7 - 2
- 1 + 2 = 7 - 4

For 1 to 8, there are these additional solutions:

- 2 + 5 = 8 - 1
- 3 + 4 = 8 - 1
- 1 + 5 = 8 - 2
- 1 + 4 = 8 - 3
- 1 + 3 = 8 - 4
- 1 + 2 = 8 - 5

The range 0 to 6 adds a few additional solutions to the original set:

- 1 + 5 = 6 - 0
- 2 + 4 = 6 - 0
- 0 + 5 = 6 - 1
- 0 + 4 = 6 - 2
- 0 + 2 = 6 - 4
- 0 + 1 = 6 - 5

Puzzle of the Week
Fill in the Blanks – 2

Using the numbers from 1 to 5 at most once, this equation has three solutions.

$$\square - \square = \square - \square$$

1 2 3 4 5

The three solutions are:

$$\boxed{3} - \boxed{1} = \boxed{4} - \boxed{2}$$

$$\boxed{4} - \boxed{2} = \boxed{5} - \boxed{3}$$

$$\boxed{4} - \boxed{1} = \boxed{5} - \boxed{2}$$

THE CHALLENGE: Use the numbers from 1 to 8 at most once to fill in these blanks.

$$\square + \square = \square + \square = \square - \square$$

1 2 3 4 5 6 7 8

EXPLORATION: Explore other number ranges. What happens if you use 1 to 7, 1 to 9, or 1 to 10? How do things change if you use 0 to 7?

Puzzle of the Week

Fill in the Blanks – 2 – Notes

THE CHALLENGE: As with the other Fill in the Blanks puzzles, a child can just play with this and eventually arrive at the answers. That exploration involves a lot of good experiences, and there is no reason to avoid it.

To be more systematic, the key observation is that the subtraction drives the solution.

For a difference of 5, the sums must be 1 + 4 and 2 + 3, and that uses up all the numbers from 1 to 4.

For a difference of 6, if the subtraction is 7 - 1 or 8 - 2, there aren't two ways of getting a sum of 6 without using a 1 (if it's 7 - 1) or a 2 (if it's 8 - 2).

So, the difference must be 7, and the last subtraction must be 8 - 1. Without using a 1, the sum of 7 can be achieved as 2 + 5 or 3 + 4, and that's our single solution.

EXPLORATION: We saw above that 1 to 7 cannot work.

Using the range 1 to 9 opens up more possibilities involving the 9.

- 9 - 1 = 8 gives 2 + 6 = 3 + 5 = 8.
- 9 - 2 = 7 gives 1 + 6 = 2 + 5 = 7
- 9 - 3 = 6 gives 1 + 5 = 2 + 4 = 6

Using the range 1 to 10 now allows us to use the 10.

- 10 - 1 = 9 gives 2 + 7 = 3 + 6 = 4 + 5 = 9
- 10 - 2 = 8 gives 1 + 7 = 3 + 5 = 8
- 10 - 3 = 7 gives 1 + 6 = 2 + 5 = 7
- 10 - 4 = 6 gives 1 + 5 = 2 + 4 = 6
- 10 - 5 = 5 gives 1 + 4 = 2 + 3 = 5

Putting 0 in the range produces quite a few surprises. There are solutions for the ranges as small as 0 to 5!

- 1 + 4 = 2 + 3 = 5 - 0
- 1 + 5 = 2 + 4 = 6 - 0
- 0 + 5 = 2 + 3 = 6 - 1
- 0 + 4 = 1 + 3 = 6 - 2
- 1 + 6 = 2 + 5 = 3 + 4 = 7 - 0
- 0 + 6 = 2 + 4 = 7 - 1
- 0 + 5 = 1 + 4 = 7 - 2
- 0 + 3 = 1 + 2 = 7 - 4

Puzzle of the Week

Fill in the Blanks – 3

Using the numbers from 1 to 5 at most once, this equation has three solutions.

$$\square - \square = \square - \square$$

1 2 3 4 5

The three solutions are:

$$\boxed{3} - \boxed{1} = \boxed{4} - \boxed{2}$$

$$\boxed{4} - \boxed{2} = \boxed{5} - \boxed{3}$$

$$\boxed{4} - \boxed{1} = \boxed{5} - \boxed{2}$$

THE CHALLENGE: Use each of the numbers from 1 to 9 at most once to fill in these blanks.

$$\square = \square + \square = \square + \square + \square$$

1 2 3 4 5 6 7 8 9

EXPLORATION: Explore other number ranges. What happens if you use 1 to 8 or 1 to 10?

Puzzle of the Week

Fill in the Blanks – 3 – Notes

THE CHALLENGE: As with the other Fill in the Blanks puzzles, a child can just play with this and eventually arrive at the answers. That exploration involves a lot of good experiences, and there is no reason to avoid it.

To be more systematic, you want to look for a driver or focus that helps reduce the search. For this puzzle, that driver is the overall sum - we need to keep it small. The smallest sum for the three numbers is 1 + 2 + 3 = 6, but that leaves the other two numbers to add up to at least 4 + 5 = 9. To balance those two things, we can add them both up and divide by two - the smallest the single number on the left can be is (1 + 2 + 3 + 4 + 5) / 2 = 7 ½. So the sum will either be 8 or 9, which we can try out individually.

If it's 8, we have 1 solution:

- 8 = 1 + 7 does not work
- 8 = 2 + 6 = 1 + 3 + 4 works!
- 8 = 3 + 5 = does not work

For 9, we have 3 solutions:

- 9 = 1 + 8 = 2 + 3 + 4 works!
- 9 = 2 +7 = 1 + 3 + 5 works!
- 9 = 3 + 6 does not work
- 9 = 4 + 5 = 1 + 2 + 6 works!

EXPLORATION: We saw above that 1 to 8 gives one solution. The range from 1 to 10 will give us many more new solutions.

- 10 = 1 + 9 = 2 + 3 + 5 works!
- 10 = 2 + 8 = 1 + 2 + 7 = 1 + 3 + 6 = 1 + 4 + 5 - 3 ways!
- 10 = 3 + 7 = 1 + 4 + 5 = 2 + 3 + 5 - 2 ways!
- 10 = 4 + 6 = 1 + 2 + 7 = 2 + 3 + 5 - 2 ways!

Puzzle of the Week

Fill in the Blanks – 4

Using the numbers from 1 to 5 at most once, this equation has three solutions.

$$\square - \square = \square - \square$$

1 2 3 4 5

The three solutions are:

$$\boxed{3} - \boxed{1} = \boxed{4} - \boxed{2}$$

$$\boxed{4} - \boxed{2} = \boxed{5} - \boxed{3}$$

$$\boxed{4} - \boxed{1} = \boxed{5} - \boxed{2}$$

THE CHALLENGE: Use each of the numbers from 1 to 8 at most once to fill in these blanks.

$$\square = \square + \square = \square + \square = \square + \square$$

1 2 3 4 5 6 7 8

EXPLORATION: Explore other number ranges. What happens if you use 1 to 9, 0 to 7, or 0 to 8?

Puzzle of the Week

Fill in the Blanks – 4 – Notes

THE CHALLENGE: As with the other Fill in the Blanks puzzles, a child can just play with this and eventually arrive at the answers. That exploration involves a lot of good experiences, and there is no reason to avoid it.

To be more systematic, use that you have three pairs of numbers that have the same sum. To make that triple sum as small as possible, we could attempt to use the numbers 1 through 6 for them. The sum of the numbers 1 through 6 is 21, and if you break that into three equal parts, that would be a sum of 7 for each individual sum.

Let's look at the two possibilities - the sum is either 7 or 8. For each number, there are only three possible ways to produce that as a sum, and we quickly find the two solutions.

7 = 1 + 6 = 2 + 5 = 3 + 4

8 = 1 + 7 = 2 + 6 = 3 + 5

EXPLORATION: Let's explore the three suggested ranges to look at.

The range 1 to 9: The only new possibilities introduced by using 1 to 9 is having 9 as the sum. That creates

9 = 1 + 8 = 2 + 7 = 3 + 6 = 4 + 5

We can select any three of those four ways to add up to 9.

The range 0 to 7: You can think of 0 to 7 as subtracting 1 from each member of the range 1 to 8. Subtracting 1 from both members of a sum will reduce the sum by 2. The sum can now be one of 5, 6, or 7.

5 = 1 + 4 = 2 + 3 - there aren't enough ways

6 = 1 + 5 = 2 + 4 - there aren't enough ways

7 = 1 + 6 = 2 + 5 = 3 + 4 - the same as before.

It turns out that the 0 doesn't help. To use 0 in a sum would force the digit to be used both in the sum and as the total, which isn't allowed.

The range 0 to 8: As we just found out for the range 0 to 7, the 0 isn't going to help. The only solution we find here is the same one we got before: 8 = 1 + 7 = 2 + 6 = 3 + 5.

Puzzle of the Week

Fill in the Blanks – 5

Using the numbers from 1 to 5 at most once, this equation has three solutions.

$$\square - \square = \square - \square$$

1 2 3 4 5

The three solutions are:

$$\boxed{3} - \boxed{1} = \boxed{4} - \boxed{2}$$

$$\boxed{4} - \boxed{2} = \boxed{5} - \boxed{3}$$

$$\boxed{4} - \boxed{1} = \boxed{5} - \boxed{2}$$

THE CHALLENGE: Use each of the numbers from 0 to 9 at exactly once to fill in these blanks.

$$\square + \square = \square + \square = \square + \square = \square + \square = \square + \square$$

0 1 2 3 4 5 6 7 8 9

EXPLORATION: Can you solve similar puzzles that break up a number range into common sums? How about four pairs using the numbers 0 to 7 or 1 to 8? How about 3 triplets from 0 to 8 or 1 to 9? How about 2 groups of 5 for the numbers from 0 to 9? Do you see any patterns for when it works and when it doesn't?

Puzzle of the Week

Fill in the Blanks – 5 – Notes

THE CHALLENGE: As with the other Fill in the Blanks puzzles, a child can just play with this and eventually arrive at the answers. That exploration involves a lot of good experiences, and there is no reason to avoid it.

If you want to be more systematic, the first question is: What is the common sum for these pairs of numbers? The five pairs have the same sum, and when we add them all up we get the same thing as adding the numbers up from 0 to 9. The sum from 0 to 9 is 45, so when we divide that by 5 we get 9 - the sum for each pair must be 9. Once that is established, the rest is simple:

0 + 9 = 1 + 8 = 2 + 7 = 3 + 6 = 4 + 5.

EXPLORATION: The first step is to see whether the sum of the range of numbers can be broken into that many equal pieces. Also, note that it makes no difference whether we start at 0 or 1, so we'll just look at starting at 0.

0 to 7 using 4 pairs: The sum of the numbers from 0 to 7 is 28. Dividing 28 into 4 pairs gives a sum of 7 for each pair. This is simple enough: 7 = 0 + 7 = 1 + 6 = 2 + 5 = 3 + 4.

0 to 2n - 1 using n pairs: After looking at 0 to 7 and 0 to 9, the pattern is clear: write n as the n possible sums.

0 to 8 using 3 triplets: The sum from 0 to 8 is 36. Dividing 36 into 3 triplets gives a sum of 12 for each triplet. The triplets will be largely driven by the three largest numbers (6, 7, 8), no two of which can be in a triplet together. This produces triplets (8, 0, 4), (7, 2, 3), and (6, 1, 5). This could also be done as (8, 1, 3), (7, 0, 5), and (6, 2, 4).

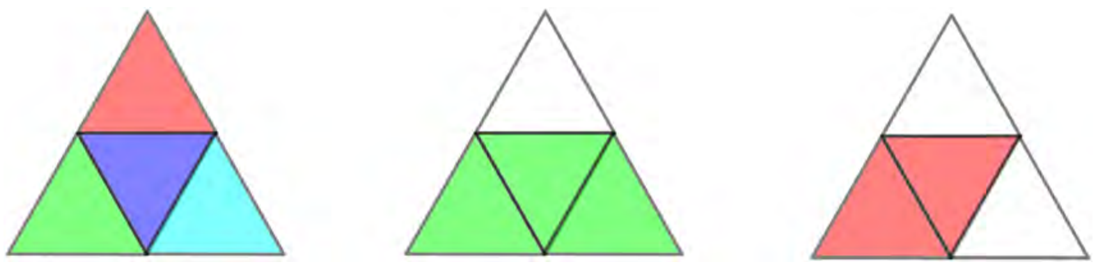
0 to 9 using 2 groups of 5: The sum from 0 to 9 is 45. 45 cannot be divided evenly into two equal groups!

The very interested child may want to go exploring further to see more examples of when this works and when it doesn't. What fun!

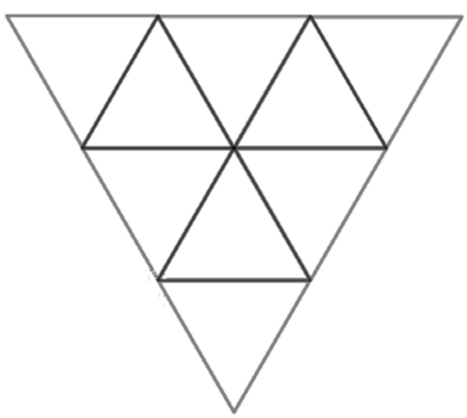
Puzzle of the Week

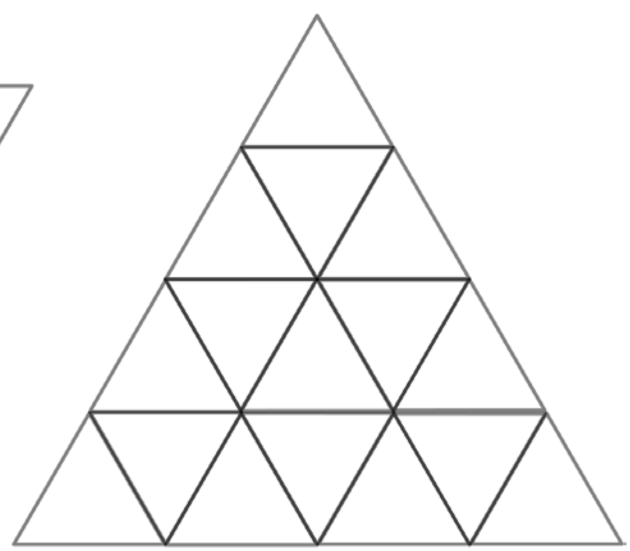
Finding the Pieces – 2

A ***trapezoid*** is a four-sided figure that has exactly one pair of parallel sides (parallel lines in a surface are lines that never meet). A ***parallelogram*** is a four-sided figure that has two pairs of parallel sides. In the large triangle on the left, there are five triangles marked – the four colored triangles and the entire triangle. The same large triangle in the middle has one of its three trapezoids colored in green. The same large triangle on the right has one of its three parallelograms colored in red.



THE CHALLENGE: In each of these figures, count the number of triangles, trapezoids, and parallelograms.





EXPLORATION: Make drawings like these for other people to count the triangles and trapezoids.

Puzzle of the Week

Picking up the Pieces – 2 – Notes

THE CHALLENGE: Work on organized counting using these two figures. The second one requires a lot of careful counting.

The large triangle on the left:

It has 9 small triangles (base 1), 3 intermediate triangles (base 2), and 1 large triangle (base 3), for a total of 13 triangles.

It has 9 small trapezoids (base 2 top 1), 3 intermediate trapezoids (base 3 top 2), and 3 large trapezoids (base 3 top 1), for a total of 15 trapezoids.

It has 9 small parallelograms (sides 1 and 1) and 6 intermediate parallelograms (sides 2 and 1), for a total of 15 parallelograms.

The large triangle on the right:

The large triangle on the right has 16 small triangles (base 1), 7 small intermediate triangles (base 2), 3 large intermediate triangles (base 3), and 1 large triangle (base 4), for a total of 27 triangles.

It has 18 small trapezoids (base 2 top 1), 9 longer small trapezoids (base 3 top 2), 3 very long small trapezoids (base 4 top 3), 9 intermediate trapezoids (base 3 top 1), 3 intermediate longer trapezoids (base 4 top 2), and 3 large trapezoids (base 4 top 1), for a total of 18 + 9 + 3 + 9 + 3 + 3 = 45 trapezoids.

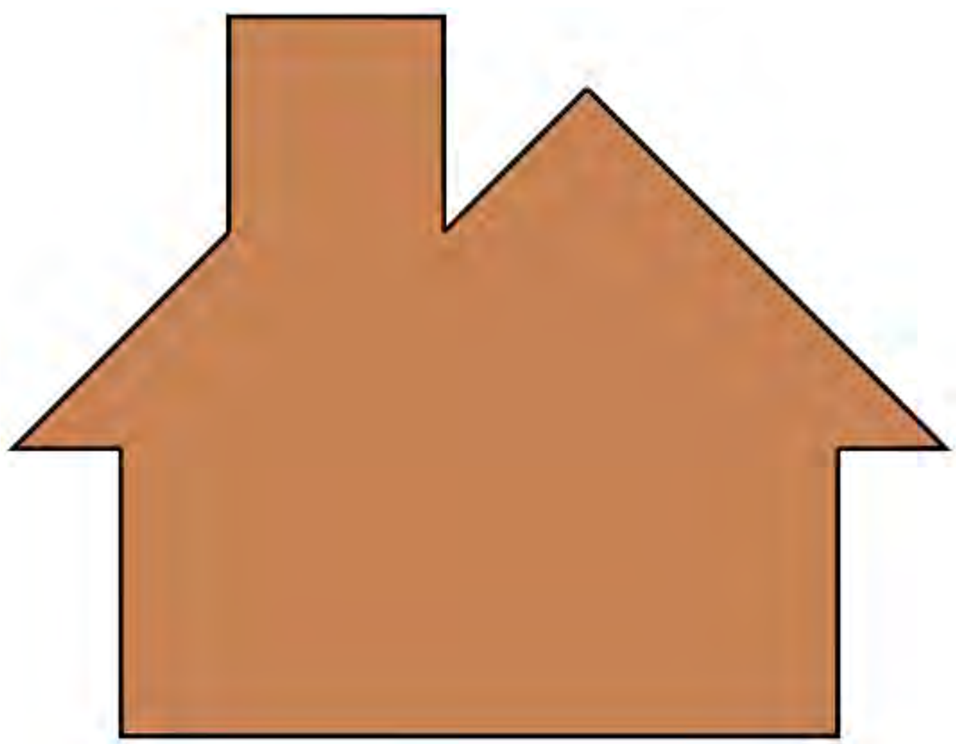
It has 18 small parallelograms (sides 1 and 1), 18 long small parallelograms (sides 2 and 1), 6 longer small parallelograms (sides 3 and 1), and 3 intermediate parallelograms (sides 2 and 2), for a total of 18 + 18 + 6 + 3 = 45 parallelograms.

Puzzle of the Week

Finding the Pieces – 3

A ***trapezoid*** is a four-sided figure that has exactly one pair of parallel sides (parallel lines in a surface are lines that never meet). A ***parallelogram*** is a four-sided figure that has two pairs of parallel sides. A ***rectangle*** is a four-sided figure with four right angles. A ***square*** is a rectangle with four equal sides. A ***right triangle*** is a triangle with a right angle.

THE CHALLENGE: Break this figure up using trapezoids, parallelograms, rectangles, squares and right triangles. Use as few pieces as you can.

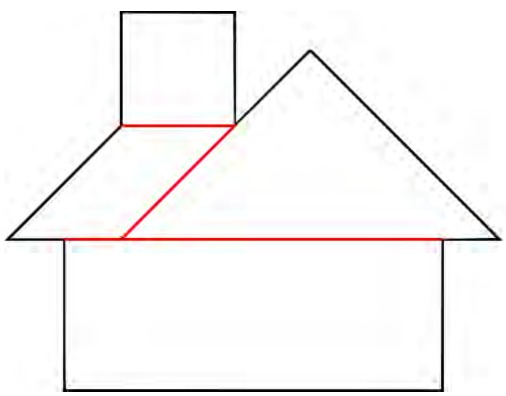


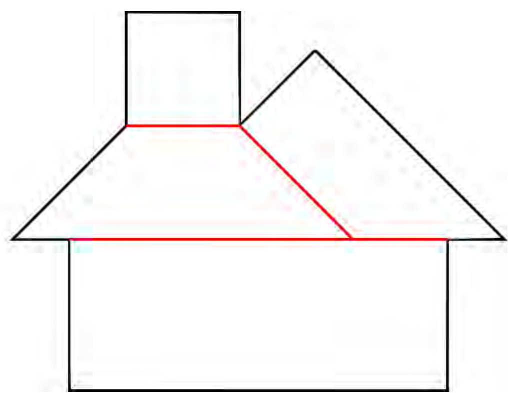
EXPLORATION: How many ways can you find to do this using this fewest number of pieces?

Puzzle of the Week

Picking up the Pieces – 3 – Notes

THE CHALLENGE: Here are two ways to break this drawing into four figures. The answer on the left uses a square, a parallelogram, a right triangle, and a rectangle. The answer on the right uses a square, two trapezoids, and a rectangle. There are other answers, such as changing the first answer by turning the square and parallelogram into a trapezoid and a right triangle. How many answers can your students find?

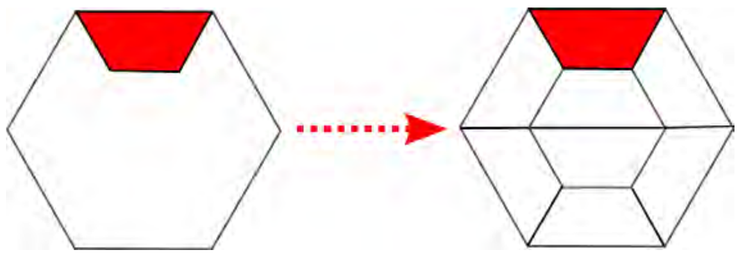




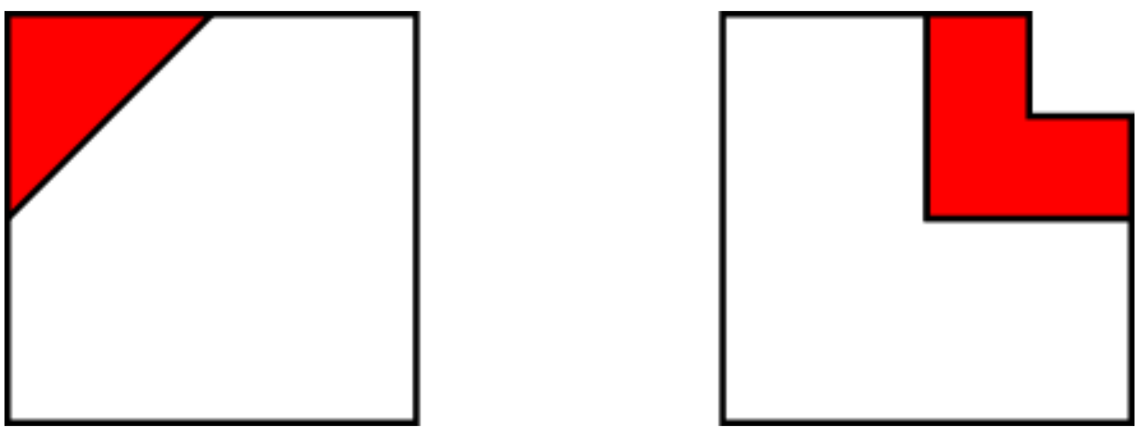
Puzzle of the Week

Finding the Pieces – 4

The figure on the left has a shaded part in red and an unshaded part. The figure on the right shows how to fill the unshaded part with seven exact copies of the shaded part.



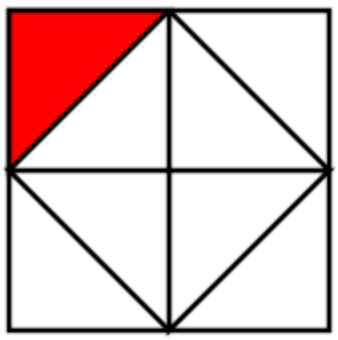
THE CHALLENGE: In these two figures, there is a shaded part and an unshaded part. Find out how many times the shaded part will exactly fit into the unshaded part.

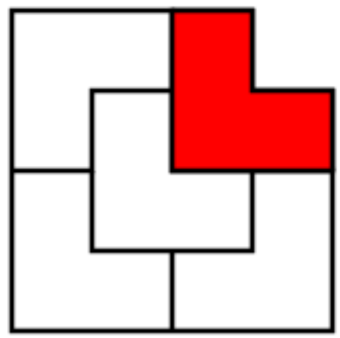


Puzzle of the Week

Finding the Pieces – 4 – Notes

THE CHALLENGE: In the left figure, there are 7 triangles like the red one. In the right figure, there are four L-shaped pieces like the red one.



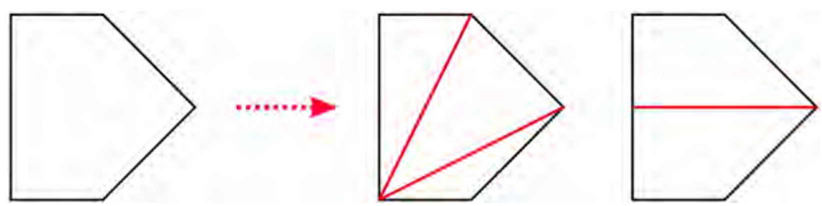


If

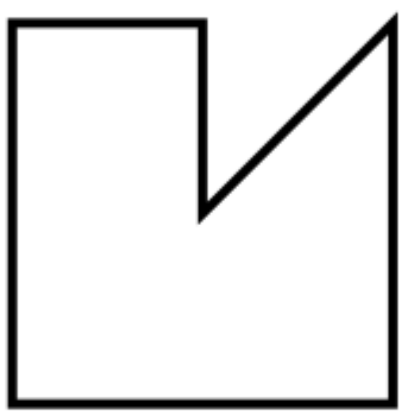
Puzzle of the Week

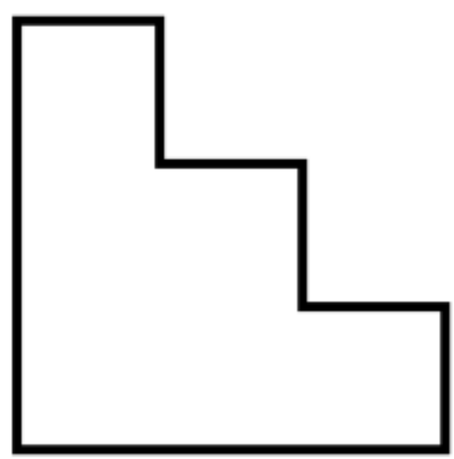
Finding the Pieces – 5

A ***trapezoid*** is a four-sided figure that has exactly one pair of parallel sides (parallel lines in a surface are lines that never meet). The figure on the left can be broken into three pieces which are triangles. It can also be broken into two trapezoids.



THE CHALLENGE: For each of these two figures, find a way to break the figure into as few triangles as possible. Also find a way to break each figure into as few trapezoids as possible.



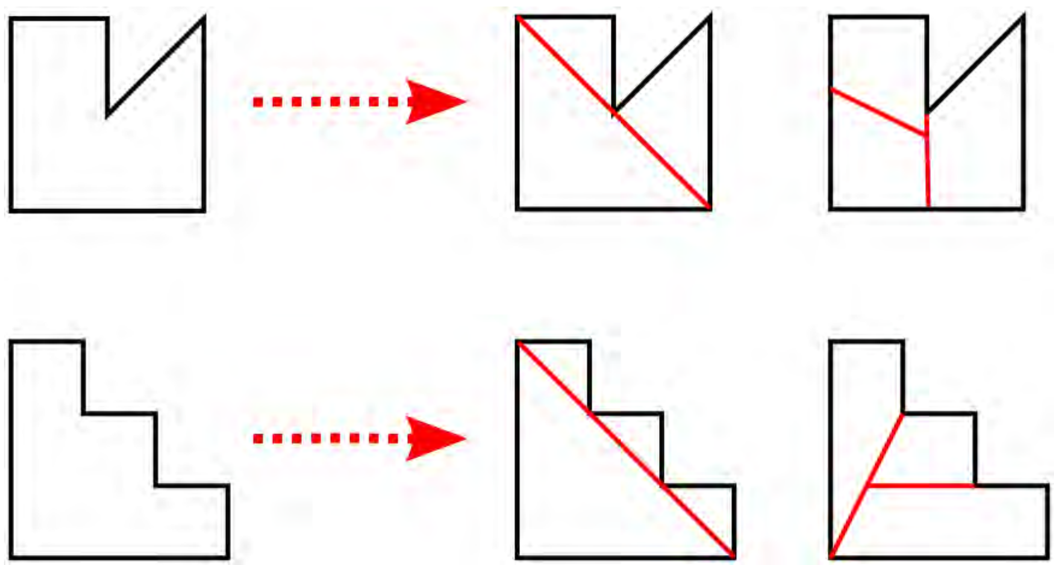


EXPLORATION: Are there other ways to break these two figures into triangles or trapezoids in as few pieces?

Puzzle of the Week

Finding the Pieces – 5 – Notes

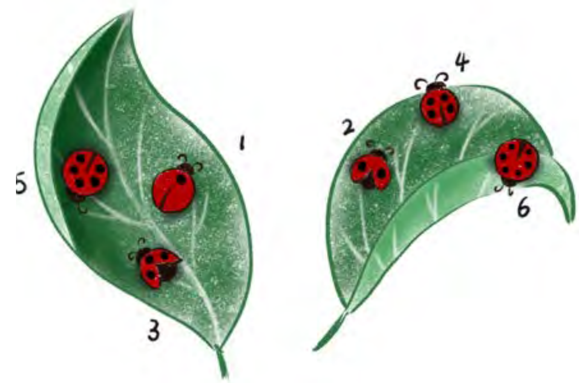
THE CHALLENGE & EXPLORATION: One example of breaking up the two figures into triangles and trapezoids is given below. There are other possible choices for how to do it, so be sure to talk about all the different possibilities that everyone finds.



Puzzle of the Week

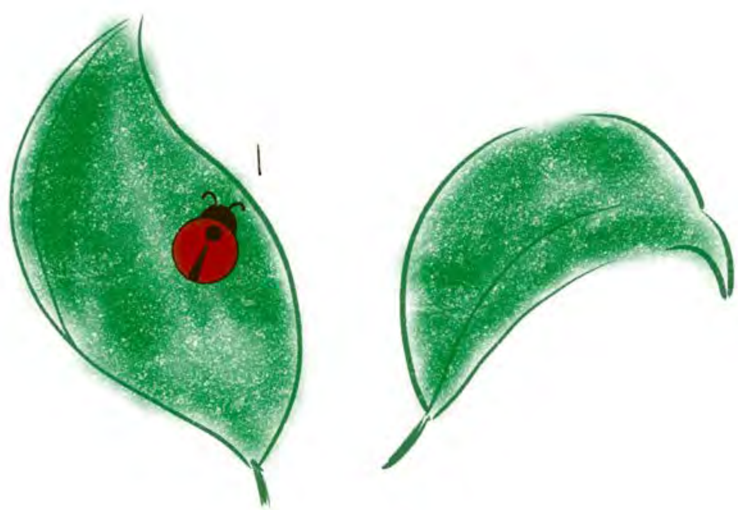
Ladybugs that don't Add Up – 1

Numbered ladybugs are landing on two leaves. The rule is: the number of dots of two ladybugs on a leaf cannot add up to the number of dots on another ladybug on that leaf. The leaf on the left is fine, but the leaf on the right has 2 + 4 = 6.



THE CHALLENGE: Starting at 1 and counting up, how high can you go putting the numbered ladybugs on two leaves while following the rule for both leaves?

EXPLORATION: How do things change if you use only even numbers? How do things change if you use only odd numbers? What are other groups of interesting numbers to look at?



Puzzle of the Week
Ladybugs that don't Add Up – 1 – Notes

THE CHALLENGE: This puzzle is all about experimenting, exploring, and keeping organized.

One general observation is that it is always safe to put all the numbers together between a number and twice that number. For example, one answer to this puzzle is (1 2) - (3 4 5 6). Powers of 2 are also safe to put together, for example (1 2 4 8) - (3 5 6 7) is an improvement.

To do this thoroughly and exhaustively requires being organized. There are five possibilities for how to place the numbers 1 to 4.

- **(1 2 4) - (3):** 5 and 6 have to go in the second group and then 8 in the first group. (1 2 4 8) - (3 5 6). 7 has to go in the second group, and then we are stuck. (1 2 4 8) - (3 5 6 7).
- **(1 2) - (3 4):** 7 must go in the first group (1 2 7) - (3 4). After that we must put 5 and 6 in the second group. (1 2 7) - (3 4 5 6). We are stuck.
- **(1 3) - (2 4):** 6 must go in the first group, and then 5 in the second. (1 3 6) - (2 4 5). We are stuck.
- **(1 4) - (2 3):** We are stuck - there is nowhere for 5.
- **(1) - (2 3 4):** 5 must go in the first group, and then we are stuck. (1 5) - (2 3 4).

The winner is (1 2 4 8) - (3 5 6 7)!

EXPLORATION: Conveniently, working with even numbers on this problem is exactly the same as working with the original numbers, only everything is doubled. So, the best answer is: (2 4 8 16) - (6 10 12 14).

Strangely enough, working with odd numbers is even easier! We can put all the odd numbers on one leaf and have no problem! The sum of two odd numbers is always an even number.

Prime numbers are tempting, but they are almost as easy as the odd numbers.

Fibonacci numbers are interesting, but your students may not know about those. Look at 1, 2, 3, 5, 8, 13, 21, 34, and so on. Any two neighboring numbers add up to the next. However, they grow quickly enough that there are no further problems. So, you can alternate leaves and not have a problem: (1 3 8 21 …) - (2 5 13 34 …).

Puzzle of the Week

Letter Substitutions – 1

Rules:

1. A letter represents a digit from 0 to 9, and has the same value throughout a single puzzle.
2. No number can start with the digit 0.
3. Within a puzzle, different letters must have different values.

$$\begin{array}{rr} & 8 \\ + & A \\ \hline B & 2 \end{array} \quad \Rightarrow \quad \begin{array}{rr} & 8 \\ + & 4 \\ \hline 1 & 2 \end{array}$$

THE CHALLENGE: Find the value of C, D, E, F and G in these puzzles.

$$\begin{array}{rr} & C \\ + & 8 \\ \hline & D \end{array} \qquad \begin{array}{rr} & E \\ + & E \\ \hline & 8 \end{array} \qquad \begin{array}{rr} & F \\ + & F \\ \hline G & 4 \end{array}$$

EXPLORATION: Make some letter substitution puzzles for your friends to solve.

Puzzle of the Week

Letter Substitutions – 1 – Notes

THE CHALLENGE: In the C + 8 = D, the sum must be less than 10. C cannot be 0 because that would break the rule of not having numbers start with 0. Therefore C is 1 and D is 9, which gives the answer: 1 + 8 = 9.

E must be half of 8, so E is 4. The answer is: 4 + 4 = 8.

This problem involves an important insight about adding: if you add two single-digit numbers, including possibly a carry, the result cannot be larger than 19, so the carry is always either 0 or 1. For this problem, the carry must be 1, so G is 1. F is half of 14, so F is 7. The answer is: 7 + 7 = 14.

EXPLORATION: Here are two, slightly more challenging, letter substitution puzzles to play with.

H + 4 = KK: K must be 1, so the problem becomes H + 4 = 11, which forces H = 7. The answer is: 7 + 4 = 11.

M + M + 8 = MN: As a carry, M could be 1 or 2. However, if M is 2 then 2 + 2 + 8 must be at least 20, which it isn't. Therefore, M is 1 and the answer becomes 1 + 1 + 8 = 10.

Puzzle of the Week

Letter Substitutions – 2

Rules:

1. A letter represents a digit from 0 to 9, and has the same value throughout a single puzzle.
2. No number can start with the digit 0.
3. Within a puzzle, different letters must have different values.

$$\begin{array}{cc} & 8 \\ + & A \\ \hline B & 2 \end{array} \quad \Rightarrow \quad \begin{array}{cc} & 8 \\ + & 4 \\ \hline 1 & 2 \end{array}$$

THE CHALLENGE: Find the value of C, D, E, F, G, and H in these puzzles.

$$\begin{array}{cc} & C \\ + & 2 \\ \hline D & E \end{array} \qquad \begin{array}{cc} & F \\ + & G \\ \hline F & H \end{array}$$

EXPLORATION: Make some letter substitution puzzles for your friends to solve.

Puzzle of the Week

Letter Substitutions – 2 – Notes

THE CHALLENGE: In problems with more letters, it is often helpful to rewrite the problem replacing each letter as you discover its value.

These problems involve an important insight about adding: if you add two single-digit numbers, including possibly a carry, the result cannot be larger than 19, so the carry is always either 0 or 1.

In the first problem, as a leading digit D cannot be 0, so it must be 1. C + 2 must be at least 10, so C is 8 or 9. If C is 9, then DE would be 11 – this would cause D and E to have the same value, which is not allowed. Therefore, C is 8, and the answer is: 8 + 2 = 10.

The second starts off the same way. F must be 1. The problem becomes 1 + G = 1H. The only way 1 + G can be 10 or higher is for G to be 9. The answer becomes: 1 + 9 = 10.

EXPLORATION: Here are three more letter substitution puzzles to play with.

J + J + K = K0: As a carry, K must be 1 or 2. If K is 1, then J + J + 1 is an odd number, which cannot end in 0. Therefore, K is 2. Now, J + J + 2 = 20 forces J to be 9. The answer is: 9 + 9 + 2 = 20.

L + L + L = M2: Three times L ends in 2 forces L to be 4 and M to be 1. The answer is: 4 + 4 + 4 = 12.

N + N + N = P4: Three times N ends in 4 forces N to be 8 and P to be 2. The answer is: 8 + 8 + 8 = 24.

Puzzle of the Week

Letter Substitutions – 3

Rules:

1. A letter represents a digit from 0 to 9, and has the same value throughout a single puzzle.
2. No number can start with the digit 0.
3. Within a puzzle, different letters must have different values.

$$\begin{array}{rr} & 8 \\ + & A \\ \hline B & 2 \end{array} \quad \Rightarrow \quad \begin{array}{rr} & 8 \\ + & 4 \\ \hline 1 & 2 \end{array}$$

THE CHALLENGE: Find the value of A, B, C, and D in these puzzles.

$$\begin{array}{rr} & A \\ & A \\ + & 6 \\ \hline B & B \end{array} \qquad \begin{array}{rr} & C \\ & C \\ + & 6 \\ \hline & D \end{array}$$

EXPLORATION: Make some letter substitution puzzles for your friends to solve.

Puzzle of the Week

Letter Substitutions – 3 – Notes

THE CHALLENGE: **For A + A + 6 = BB**: The largest number B + B + 6 can be is 9 + 9 + 6 = 24, so B is either 1 or 2. A + A + 6 must be an even number, so B = 2. A + A + 6 = 22 means A + A = 16, so A = 8.

So, the answer is 8 + 8 + 6 = 22.

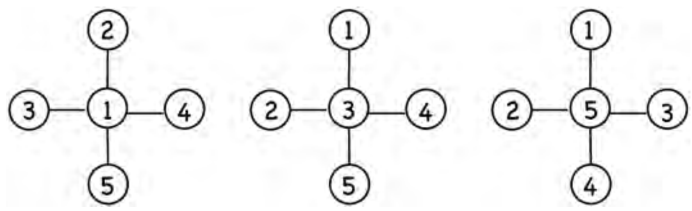
C + C + 6 = D: The only way that C + C + 6 can be less than 10 is if C is 1.

The answer is 1 + 1 + 6 = 8.

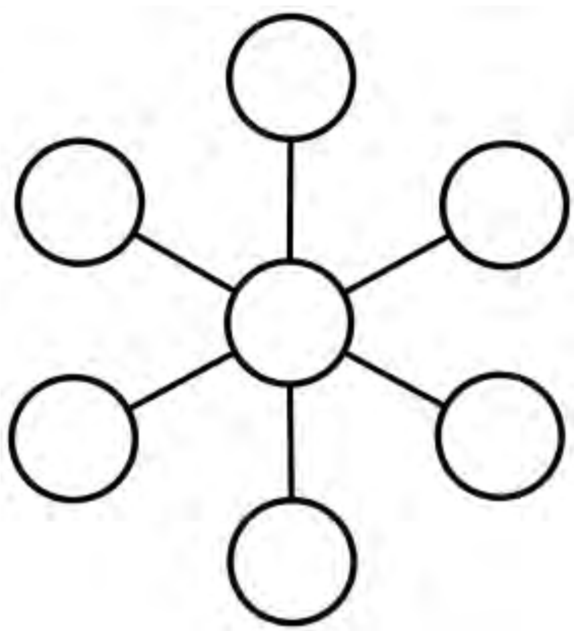
Puzzle of the Week

Magic Flowers – 1

The sums in a *Magic Flower* are the same for all straight lines. These Magic Flowers use numbers from 1 to 5.



THE CHALLENGE: Use the numbers from 1 to 7 to make Magic Flowers.



1 2 3 4 5 6 7

EXPLORATION: Play around with what happens with Magic Flowers with more than three lines.

Puzzle of the Week

Magic Flowers – 1 – Notes

THE CHALLENGE: Use this puzzle after the Magic Pluses puzzle.

This, like many of these Puzzles of the Week, can be attacked by playing around with the numbers until a solution is found. Don't be tempted to think that a more structured approach is better for your students – a great deal will be learned about the mathematics involved and about problem solving by tenaciously working through many examples. Finding a solution, by any method, is always a wonderful reward.

Most of these "equal sum" puzzles can be attacked by adding up some of the straight lines. In the case of this puzzle, add up the three directions - this will include all the numbers once, plus the central circle's number two extra times. The sum of 1 to 7 is 28. So, the sum of the three lines is 28 + 2x1, 28 + 2x2, 28 + 2x3, 28 + 2x4, 28 + 2x5, 28 + 2x6, or 28 + 2x7. Of those, only 30 = 28 + 2x1, 36 = 28 + 2x4, and 42 = 28 + 2x7 are divisible by 3. Dividing them by 3 tells us that the common sums are either 10 = 30 / 3, 12 = 30 / 3, or 14 = 30 / 3.

Let's look at those three cases.

Common Sum of 10: The central circle will be 1. Making a sum of 10 with a 1 in the center means the other two numbers add up to 9. So, the three directions are: (2 1 7) - (3 1 6) - (4 1 5).

Common Sum of 12: The central circle will be 4. Making a sum of 12 with a 4 in the center means the other two numbers add up to 8. So, the three directions are: (1 4 7) - (2 4 6) - (3 4 5).

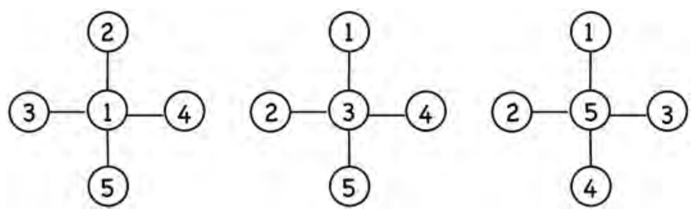
Common Sum of 14: The central circle will be 7. Making a sum of 14 with a 7 in the center means the other two numbers add up to 7. So, the three directions are (1 7 6) - (2 7 5) - (3 7 4).

EXPLORATION: Look at the Notes page for Magic Flowers - 2.

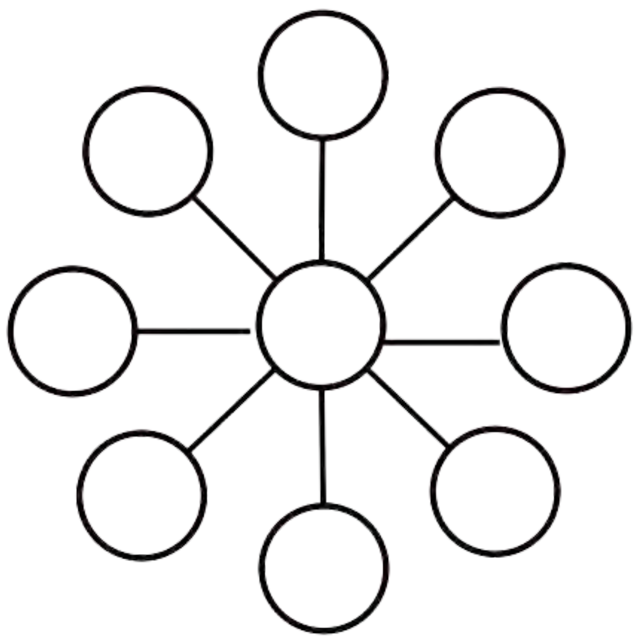
Puzzle of the Week

Magic Flowers – 2

The sums in a *Magic Flower* are the same for all straight lines. These Magic Flowers use numbers from 1 to 5.



THE CHALLENGE: Use the numbers from 1 to 9 to make Magic Flowers.



1 2 3 4 5 6 7 8 9

EXPLORATION: Looking at the solutions for Magic Flowers 1 and 2, what do you expect the solutions to be for Magic Flowers with even more lines?

Puzzle of the Week

Magic Flowers – 2 – Notes

THE CHALLENGE: This, like many of these Puzzles of the Week, can be attacked by playing around with the numbers until a solution is found. Don't be tempted to think that a more structured approach is better for your students – a great deal will be learned about the mathematics involved and about problem solving by tenaciously working through many examples. Finding a solution, by any method, is always a wonderful reward.

Most of these "equal sum" puzzles can be attacked by adding up some of the straight lines. In the case of this puzzle, add up the four directions - this will include all the numbers once, plus the central circle's number three extra times. The sum of 1 to 9 is 45. So, the possible sums of the three lines are 45 + 3x1 through 45 + 3x9. Of those, only 48 = 45 + 3x1, 60 = 45 + 3x5, and 72 = 45 + 3x9 are divisible by 4. Dividing them by 4 tells us that the common sums are either 12 = 48 / 4, 15 = 60 / 4, or 18 = 72 / 4.

Let's look at those three cases.

Common Sum of 12: The central circle will be 1. Making a sum of 12 with a 1 in the center means the other two numbers add up to 11. So, the four directions are: (2 1 9) - (3 1 8) - (4 1 7) - (5 1 6).

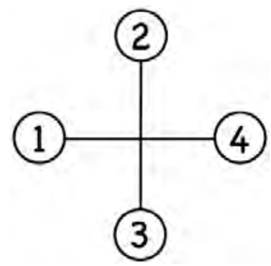
Common Sum of 15: The central circle will be 5. Making a sum of 15 with a 5 in the center means the other two numbers add up to 10. So, the four directions are: (1 5 9) - (2 5 8) - (3 5 7) - (4 5 6).

Common Sum of 18: The central circle will be 9. Making a sum of 18 with a 9 in the center means the other two numbers add up to 9. So, the four directions are (1 9 8) - (2 9 7) - (3 9 6) - (4 9 5).

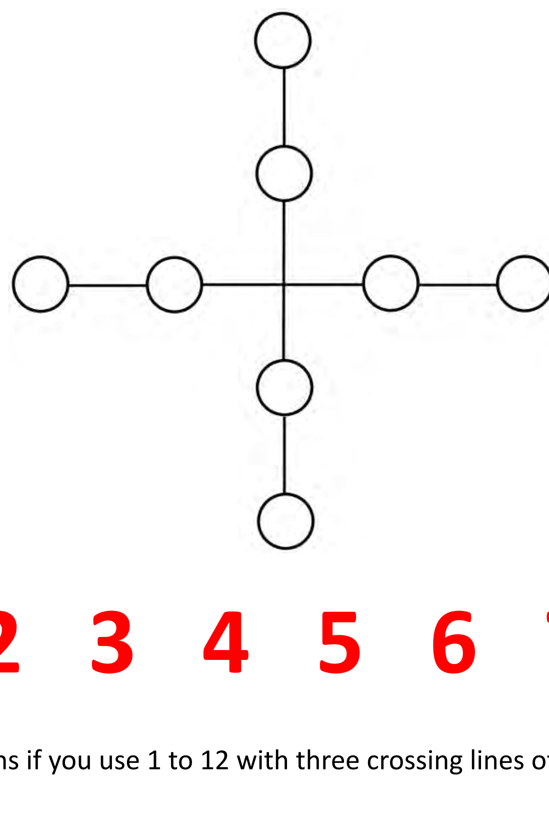
EXPLORATION: For numbers that go from 1 to 2n-1, the central circle has either 1, n, or 2n-1. The sums will be 2n+2 = 1 + 2 + 2n-1, 3n = 1 + n + 2n-1, and 4n-2 = 1 + 2n-2 + 2n-1.

Puzzle of the Week
Magic Pluses

A *Magic Plus* is a plus sign with all the sums the same. This one uses the numbers from 1 to 4.



THE CHALLENGE: Make a Magic Plus with the numbers from 1 to 8.



1 2 3 4 5 6 7 8

EXPLORATION: What happens if you use 1 to 12 with three crossing lines of four circles? Play around with other configurations of circles.

Puzzle of the Week
Magic Pluses – Notes

THE CHALLENGE: Use this as the first “equal sum” puzzle. The Magic Flowers puzzles are slightly harder than this one.

This, like many of these Puzzles of the Week, can be attacked by playing around with the numbers until a solution is found. Don’t be tempted to think that a more structured approach is better for your students – a great deal will be learned about the mathematics involved and about problem solving by tenaciously working through many examples. Finding a solution, by any method, is always a wonderful reward.

To approach this in a more structured way, start by calculating what the sums must be. The sum of the numbers from 1 to 8 is 36. Breaking this into two equal groups will mean each group has a sum of 18. At this point, there are lots of ways to do this and they all work. If you take any group of numbers that adds up to 18, the remaining numbers will as well.

The solutions are:

- (1 2 7 8) - (3 4 5 6)
- (1 3 6 8) - (2 4 5 7)
- (1 4 5 8) - (2 3 6 7)
- (1 4 6 7) - (2 3 5 8)

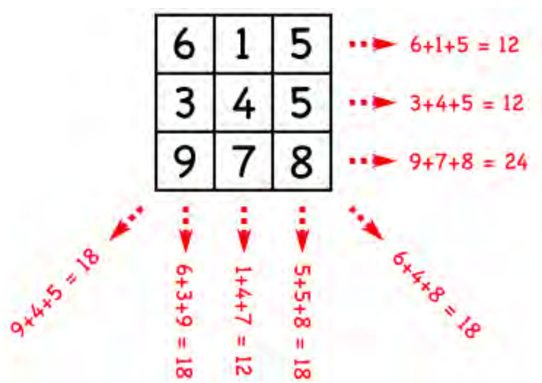
EXPLORATION: The analysis for using 1 to 12 is the same as before. The sum of 1 to 12 is 78. Breaking that into three equal parts gives a sum of 26 in each direction. There are a lot of solutions. Here are a few:

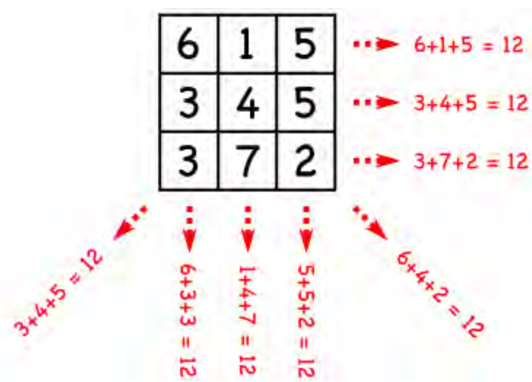
- (1 2 11 12) - (3 4 9 10) - (5 6 7 8)
- (1 2 11 12) - (3 5 8 10) - (4 6 7 9)
- (1 3 10 12) - (2 5 8 11) - (4 6 7 9)

Puzzle of the Week

Magic Squares – 1

In a *Magic Square*, all the rows, columns and diagonals add up to the same number. This first square is <u>not</u> a Magic Square. The second one is a Magic Square with a constant sum of 12.





THE CHALLENGE: Use each of the numbers 3, 5, 6, and 9 once to complete this Magic Square.

8	1	
		7
4		2

3 5 6 9

Puzzle of the Week

Magic Squares – 1 – Notes

THE CHALLENGE: This is meant to be an introductory warmup puzzle for Magic Squares, so it doesn't require much careful analysis. This, like many of these Puzzles of the Week, can be attacked by playing around with the numbers until a solution is found. Don't be tempted to think that a more structured approach is better for your students – a great deal will be learned about the mathematics involved and about problem solving by tenaciously working through many examples. Finding a solution, by any method, is always a wonderful reward.

Looking at the upper right corner, we know that the common sum is equal to that corner plus 9 more (looking at its row and its column). Considering the diagonal the upper right corner is on, we know that the other two entries on that diagonal add up to 9. So the central square must be 5.

If the central square is 5, then we have a diagonal of (8 5 2), whose sum is 15. Now we've got the common sum.

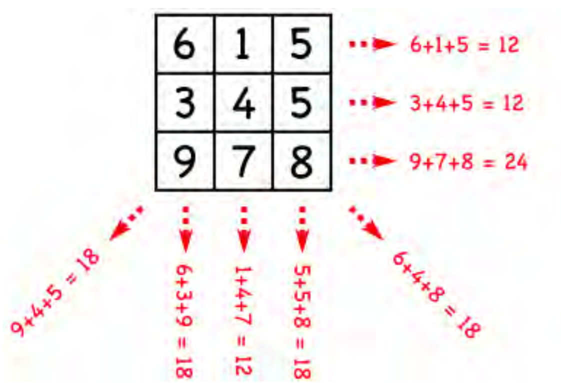
In the bottom row, 15 = 4 + (middle square) + 2 tells us the middle square of the bottom row is 9. We can continue in this way now that we know the common sum.

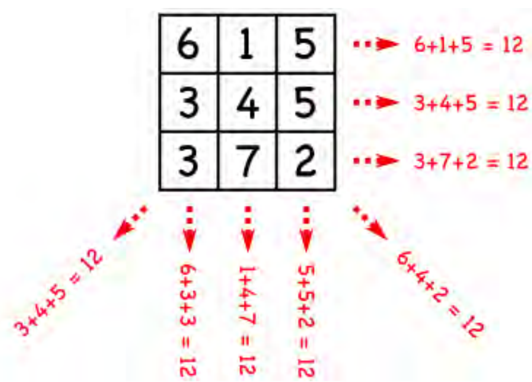
The final solution (by rows) is: (8 1 6) (3 5 7) (4 9 2).

8	1	6
3	5	7
4	9	2

Puzzle of the Week
Magic Squares – 2

In a *Magic Square*, all the rows, columns and diagonals add up to the same number. This first square is <u>not</u> a Magic Square. The second one is a Magic Square with a constant sum of 12.





THE CHALLENGE: Use each of the numbers 1, 2, 4, 7, and 8 once to complete this Magic Square.

	9	
	5	3
6		

1 2 4 7 8

Puzzle of the Week

Magic Squares – 2 – Notes

THE CHALLENGE: This is meant to be an intermediate warmup puzzle for Magic Squares, so it doesn't require much careful analysis. This, like many of these Puzzles of the Week, can be attacked by playing around with the numbers until a solution is found. Don't be tempted to think that a more structured approach is better for your students – a great deal will be learned about the mathematics involved and about problem solving by tenaciously working through many examples. Finding a solution, by any method, is always a wonderful reward.

The simplest way to start analyzing this puzzle is to find the common sum. Each row adds up to the common sum. Also, the three rows contain the numbers from 1 to 9 and add up to three times the common sum. Therefore, three times the common sum is 45 (the sum of 1 to 9), so the common sum is 15.

After that, start with the lines that already have two numbers and fill in the missing numbers to make them all add up to 15.

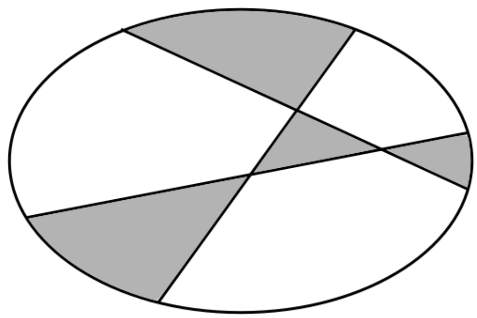
The final solution (by rows) is: (2 9 4) (7 5 3) (6 1 8).

2	9	4
7	5	3
6	1	8

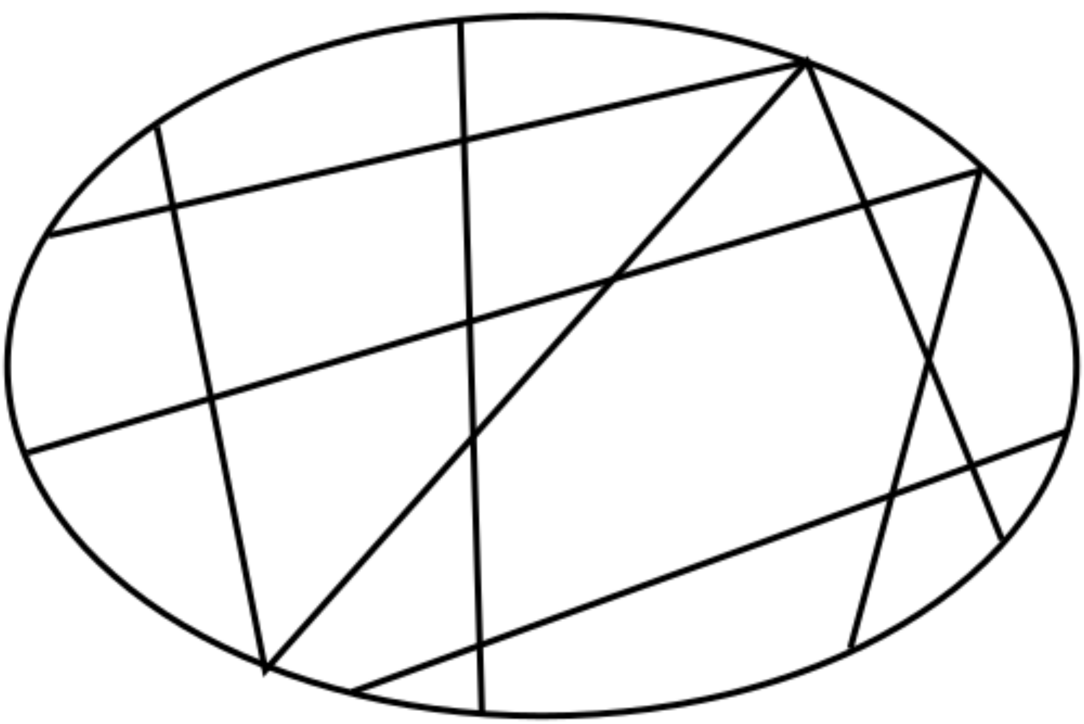
Puzzle of the Week

Map Coloring with 2 Colors – 1

Map makers color maps so regions sharing a border have different colors. Here is a map where the regions are created by drawing straight lines across the map. Notice how it has been colored using just two colors.



THE CHALLENGE: Color this more complicated map using just two colors.



Puzzle of the Week

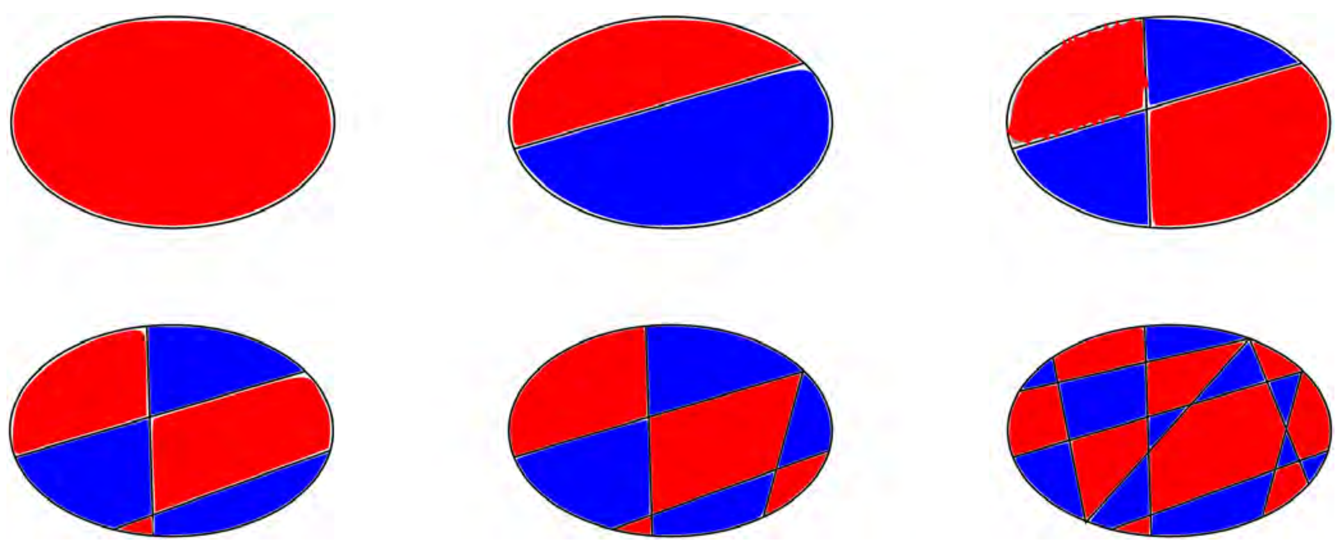
Map Coloring with 2 Colors – 1 – Notes

THE CHALLENGE: Here is a fun demonstration for coloring maps such as the two on the first page, where the regions are made by a collection of lines. It's probably best to do this on a whiteboard or some other easily erasable surface.

Start with an empty oval. Now draw a line across it. Lightly darken one side of the line.

Next, draw another line across that. Flip the color used for every region on one side of the line - if it was white, make it dark, and if it was dark, make it light.

Keep adding lines one at a time and flipping all the colors on one side of the line. Do this for as long as your patience holds out. In the end you will have a map that is colored using just two colors!

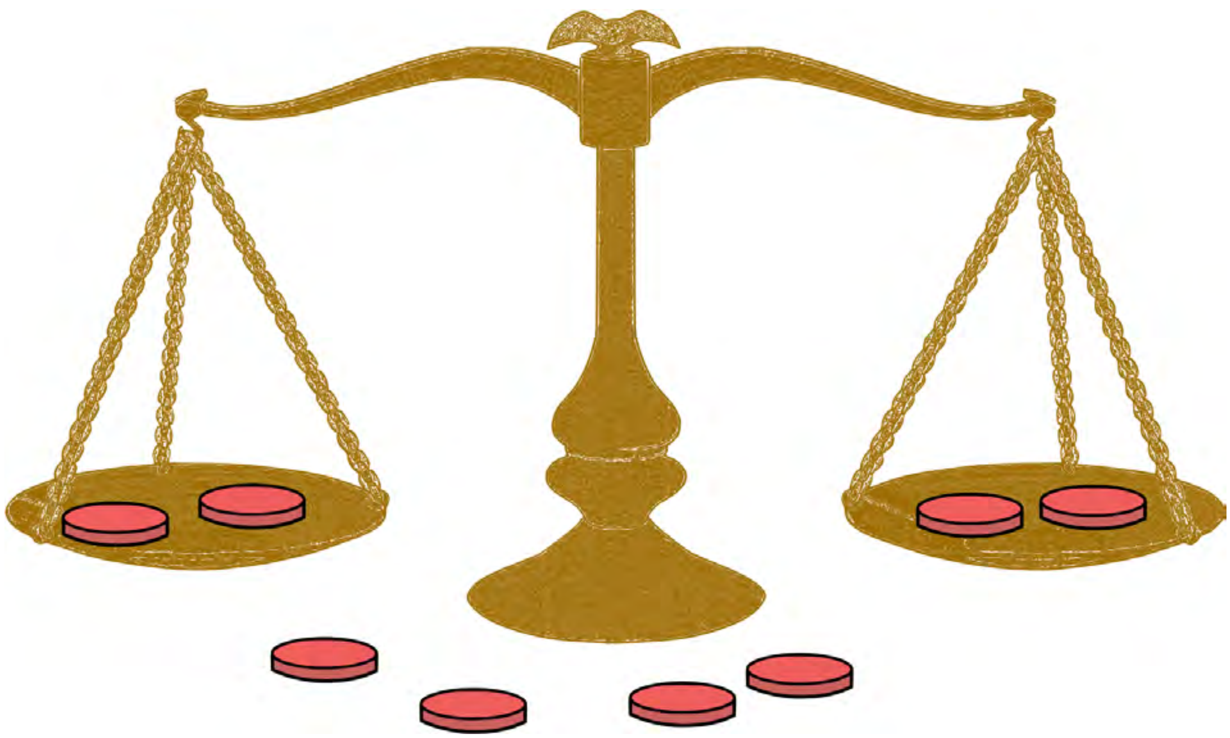


Puzzle of the Week

Pan Balance – 3

A pan balance tells you when its two sides are carrying the same amount of weight or whether one side is heavier than the other.

THE CHALLENGE: You have 25 coins. All the coins but one weigh the same amount. The remaining coin is a counterfeit and is just a tiny bit lighter or heavier (you don't know which). Using just two weighings, determine whether the counterfeit is lighter or heavier.



EXPLORATION: How does your strategy change if you start with a different number of coins? Is two weighings always enough? Is the strategy different for different numbers?

Puzzle of the Week

Pan Balance – 3 – Notes

THE CHALLENGE: Start by forming two groups of coins of equal size 7 (8 will also work). Weigh these two groups against each other.

If the two groups balance, then the counterfeit must be in the remaining group of coins that has size 11. Put together any group of 11 coins from the first two groups and weigh those against the remaining coins.

If the two groups do not balance, then all the remaining coins must be normal. Pick any 7 of the remaining coins and weigh them against either of the two original groups. If they balance, then the counterfeit was in the other group. If they don't balance, then the counterfeit was in that group from the original two groups. Either way, you will then know whether it was lighter or heavier.

EXPLORATION: This approach will always work. Start by forming two groups of equal size x. Suppose this leaves out y coins. To be able to follow the procedure described above, x + x must be at least as big as y (in case the two groups balance), and y must be at least as big as x (in case they don't balance). This is easily achieved for any number other than five coins by letting x be the original number of coins divided by four and rounding up if it doesn't divide evenly.

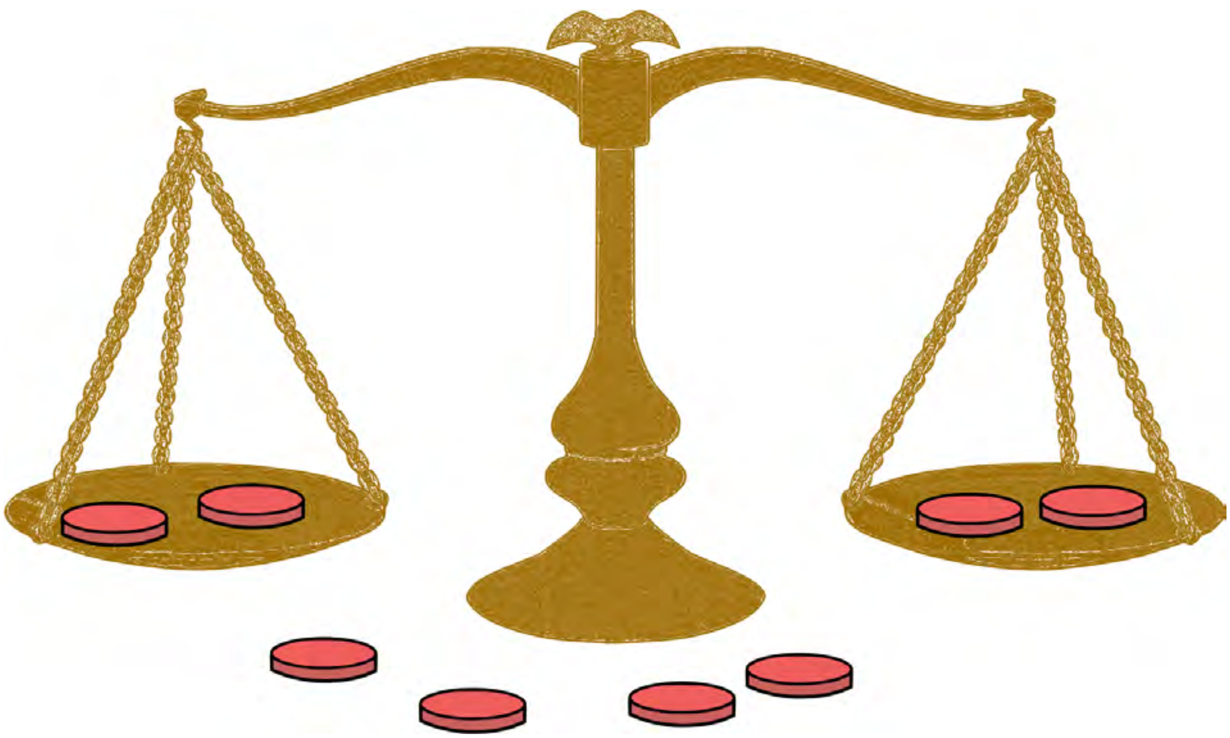
Surprisingly, I believe that five coins requires three weighings!

Puzzle of the Week

Pan Balance – 4

A pan balance tells you when its two sides are carrying the same amount of weight or whether one side is heavier than the other.

THE CHALLENGE: You have 6 coins. Four of the coins weigh the same amount, and the remaining two coins are counterfeits. Identify the two counterfeits and their weight types using just three weighings.



EXPLORATION: Investigate what happens to your method if there are more normal coins.

Puzzle of the Week

Pan Balance – 4 – Notes

THE CHALLENGE: Start by forming two groups of three coins and weigh these two groups against each other.

If they balance, then there must be one light coin in each group. For each group of three, do the following. Weigh two of the coins in the group of three - if they balance, the remaining coin is the light one; if they don't balance, the lighter coin is the light one.

If they don't balance, then both light coins must be in the lighter group of three. Weigh two of the lighter group coins against each other - if they balance, they are both lighter coins; if they don't balance, then the lighter one and the remaining coin are the two lighter coins.

EXPLORATION: This method is dependent on having groups of three. So, for more than six coins we will need more than two weighings.

Puzzle of the Week

Parentheses – 1

Where you choose to put parentheses can change the value of an expression.

For example $5 - 3 + 1$ can become either $5 - (3 + 1) = 1$ or $(5 - 3) + 1 = 3$.

THE CHALLENGE: Find the places to put parentheses in this expression to make its value 6.

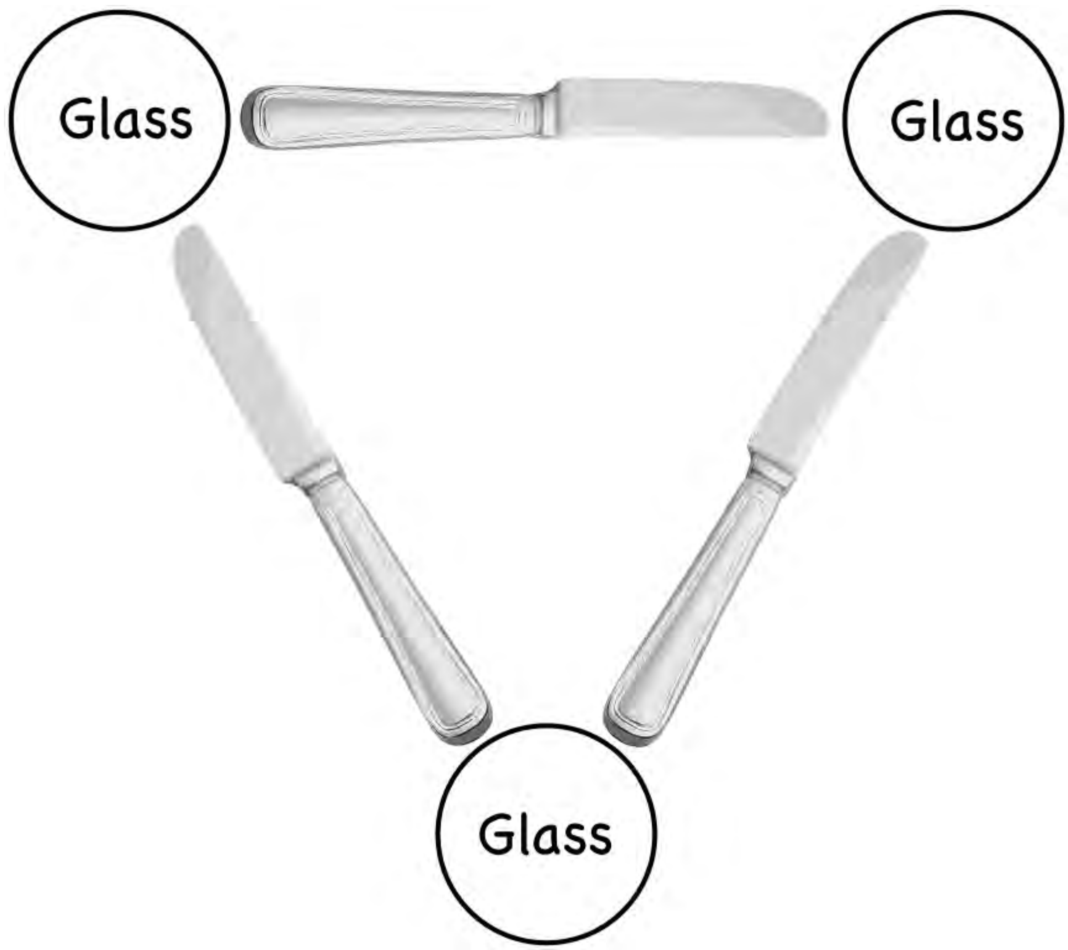
$$10 - 3 - 5 - 2 + 8 - 4$$

EXPLORATION: Make some of these puzzles for your friends. Have you found any examples where you can put in the parentheses in two different ways and still get the right answer?

Puzzle of the Week

Bridges with Knives

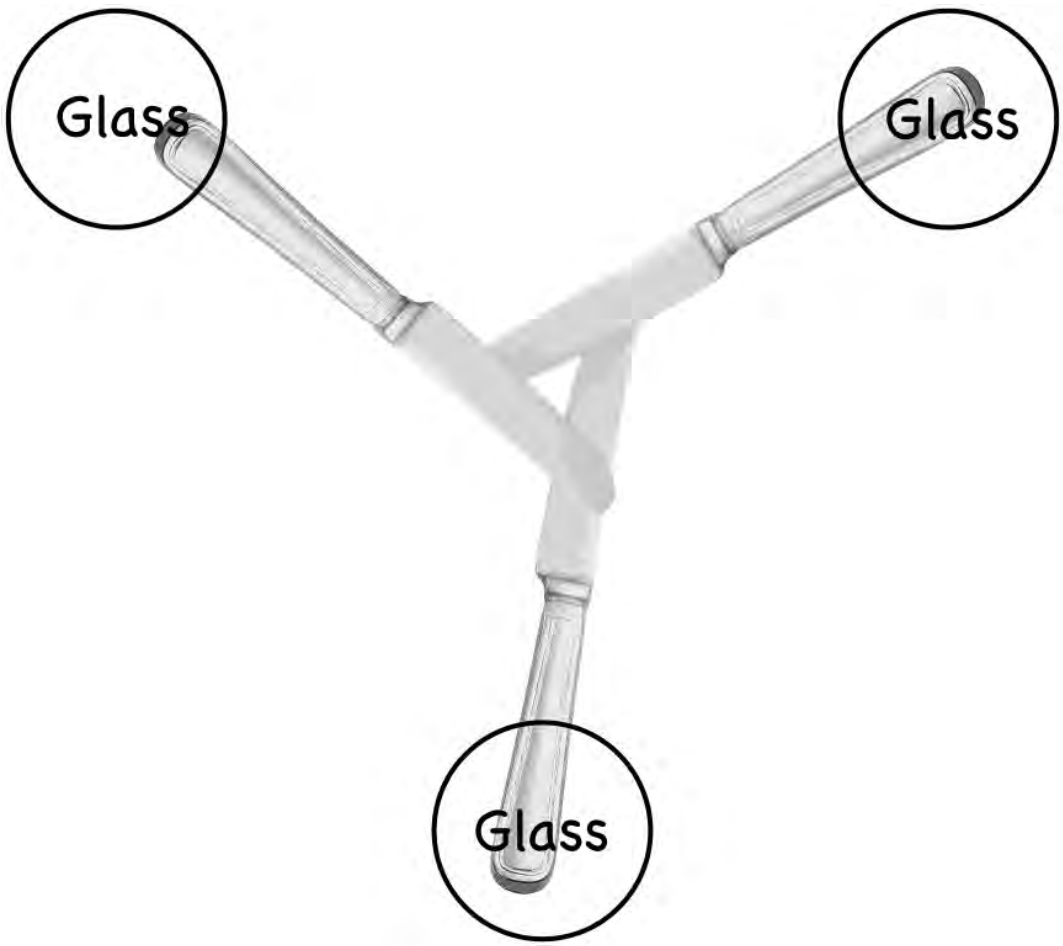
THE CHALLENGE: You have three identical water glasses and three identical regular dinner knives. Place the three glasses so their rims are just slightly farther apart than the length of a knife. Your challenge is, without moving the glasses, to place the knives on the rims of the three glasses so that they form a solid structure capable of supporting a salt shaker at any point (if it could balance) and which connects the three glasses.



Puzzle of the Week

Bridges with Knives – Notes

THE CHALLENGE: The key is to interlace the blades of the three knives in the center between the glasses. The handles will rest on the rims of the glasses, and the blades will meet in a weave pattern that is mutually supportive. The illustration below is not quite right - the tip of each knife blade should rest on top of the next knife. Try it - this structure is surprisingly strong!

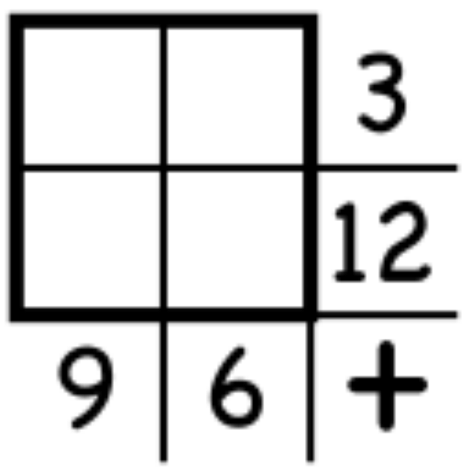


Puzzle of the Week

Square Sums – 1

The rows and columns add up to the numbers on the outside of this 2 by 2 square.

THE CHALLENGE: Using the numbers from 1 to 7, solve for the missing numbers in this square.



1 2 3 4 5 6 7

EXPLORATION: Make a Square Sum challenge for someone else.

Puzzle of the Week

Square Sums – 1 – Notes

THE CHALLENGE: Using the smallest and largest numbers helps to narrow down the possibilities.

The only two numbers that add up to 12 are 5 and 7, so they must go along the bottom row.

The only two numbers that add up to 3 are 1 and 2, so they must go along the top row.

The only way to get 6 using 5, 7, 1, and 2 is to add 5 and 1, so they must be in the rightmost column.

At this point, we have the solution: (7 5 12) (2 1 3) (9 6 +).

EXPLORATION: It is easy to make these puzzles. Start by putting four numbers on the inside, find their sums, and then create the puzzle leaving out the four numbers on the inside.

While creating puzzles that way is easy, they aren’t always the most fun to solve. To increase the fun, add some restrictions. For example, say that each number is used at most once. Also, restrict the possible numbers by saying they are in a range or that they all have some characteristic, such as all being odd.

Puzzle of the Week

Stick Areas

Suppose you have 12 sticks to use to enclose a region with horizontal and vertical sides. These three possible ways for doing that enclose areas of size 3, 5, and 9.

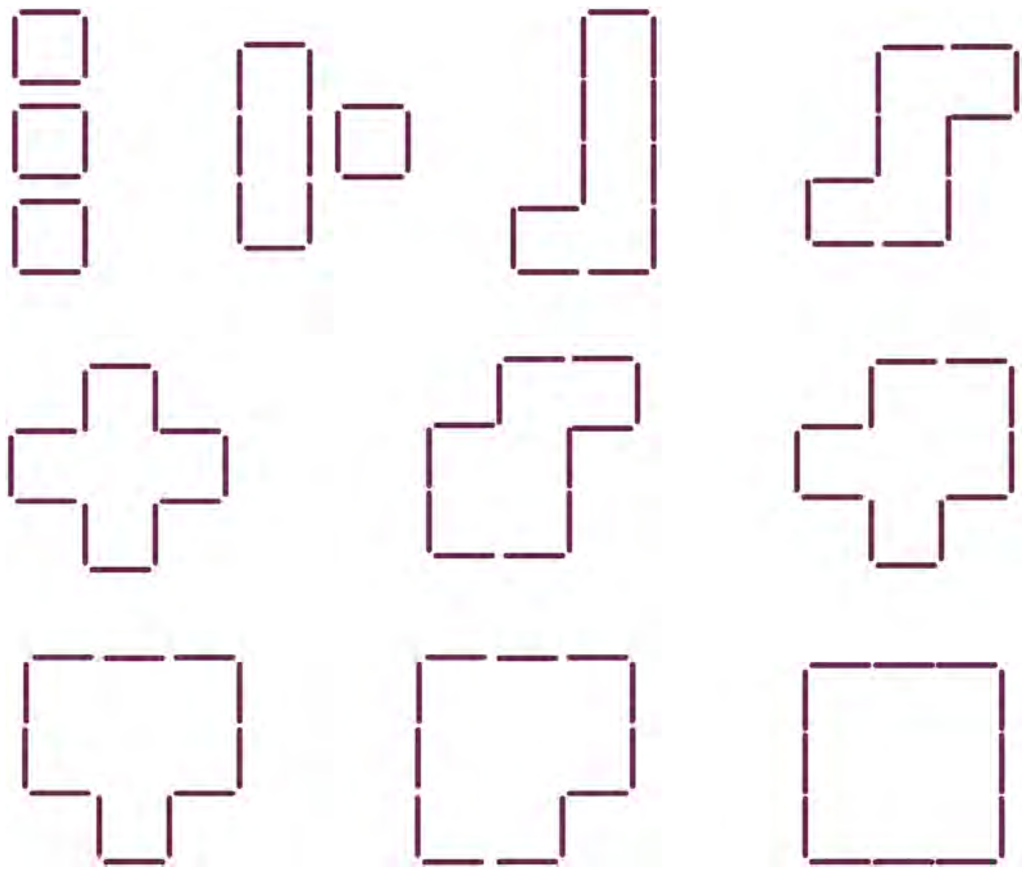
THE CHALLENGE: Find all the possible areas that you can create with 12 sticks.

EXPLORATION: What happens when you change the number of sticks? Are there some numbers of sticks that don't work at all?

Puzzle of the Week

Stick Areas – Notes

THE CHALLENGE: Here are some examples for creating areas from 3 to 9.



EXPLORATION: The sticks can always be paired up, so only even numbers of sticks will work. For the even numbers, two kinds of things can happen.

Multiples of 4: These numbers will behave just as 12 did. For a number of the form 4n, the smallest area will be produced by n 1 by 1 squares, and the largest by an n by n square.

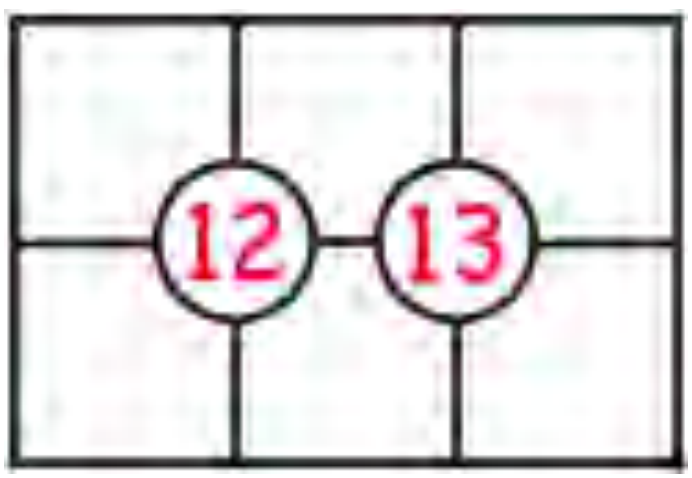
Multiples of 4 plus 2: These numbers will be similar to a multiple of four, only they will have an extra bump. For a number of the form 4n + 2, the smallest area will be produced by (n - 1) 1 by 1 squares plus one 2 by 1 rectangle. The largest area will be an n by (n + 1) rectangle.

Puzzle of the Week

Sujiko Puzzle – 1

In this small 2 by 3 Sujiko puzzle, use each of the numbers from 1 to 6 once in the six squares. The number in each circle must be the sum of the four squares that surround it.

THE CHALLENGE: Fill in this Sujiko puzzle.



1 2 3 4 5 6

EXPLORATION: How many different answers can you find?

Puzzle of the Week

Sujiko Puzzle – 1 – Notes

THE CHALLENGE & EXPLORATION: Each column of numbers - the 2 and 6, the 1 and 3, and the 4 and 5 - can be reversed in the answer given below. This produces a collection of 2 x 2 x 2 = 8 possible answers, one of which is shown below.

The 1 and 3 must be in the middle column. Call the middle numbers A and B. Add things up in two ways. The sum of all the entries in the two 2 by 2 squares is 12 + 13 = 25. On the other hand, that sum is all six numbers (1 to 6) with the middle column counted twice - that is, the sum is 1 + 2 + 3 + 4 + 5 + 6 + A + B = 21 + A + B. This means 25 = 21 + A + B, or 4 = A + B. This forces A and B to be 1 and 3.

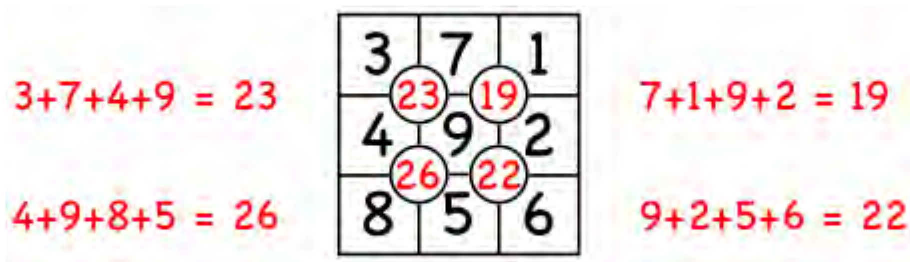
With the middle column being 1 and 3, that forces the sum of the two entries in the right column to be 9. We can only get 9 as 4 + 5 or 3 + 6, but the 3 is already used. Therefore the rightmost column is 4 and 5.

There are only two numbers, 2 and 6, remaining for the leftmost column, and they work.

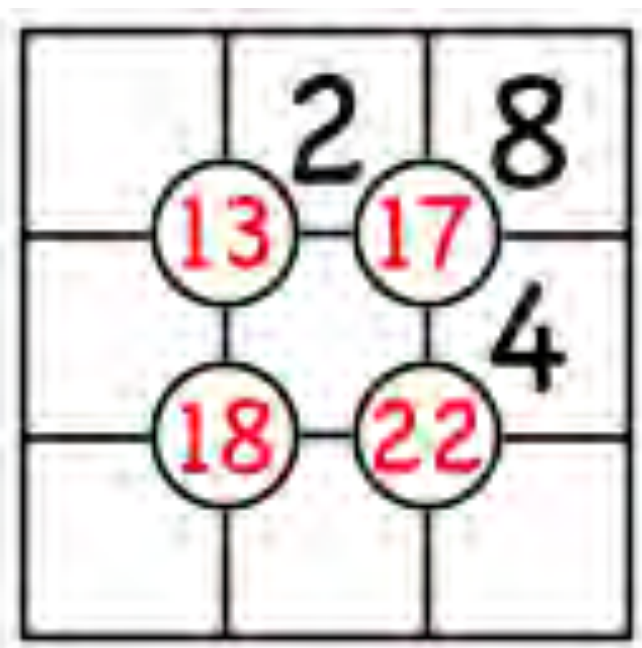
Puzzle of the Week

Sujiko Puzzle – 2

In a Sujiko puzzle, use each of the numbers from 1 to 9 once in the nine squares. The number in each circle must be the sum of the four squares that surround it.



THE CHALLENGE: Fill in this Sujiko puzzle.



1 3 5 6 7 9

Puzzle of the Week

Sujiko Puzzle – 2 – Notes

THE CHALLENGE: The upper right 2 by 2 corner has three of the four squares filled in, so that is the place to start. The three squares filled in, (2 8 4), add up to 14. To make the sum of the four squares equal to 17 (in the circle), we need the central square to be 17 - 14 = 3.

The smallest and largest numbers are often good places to start. For the bottom right 2 by 2 corner, the two numbers we have, (3 4), plus the two missing numbers must add up to 22. So, the missing two numbers on the right side of the bottom row add up to 15. We can get 15 as the sum of 6 + 9 or 7 + 8. However, the 8 is not available, so they must be 6 + 9.

Let's look at the bottom left 2 by 2 corner. On the right side of that corner we will either have 3 + 6 or 3 + 9. If it were 3 + 6, we would need two more numbers that add up to 9 and those are not available. So it must be 3 + 9. The remaining two numbers in that bottom left corner must add up to 6, and the only possible way to do that is with 1 + 5.

The only unused number at this point is the 7, so it must go in the upper left corner.

Three of the four numbers in the upper left 2 by 2 corner are (7 2 3), so the remaining number must be 1.

At this point we have the complete solution! Here it is row by row:

(7 2 8)

(1 3 4)

(5 9 6)

Puzzle of the Week

Sujiko Puzzle – 3

In a Sujiko puzzle, use each of the numbers from 1 to 9 once in the nine squares. The number in each circle must be the sum of the four squares that surround it.

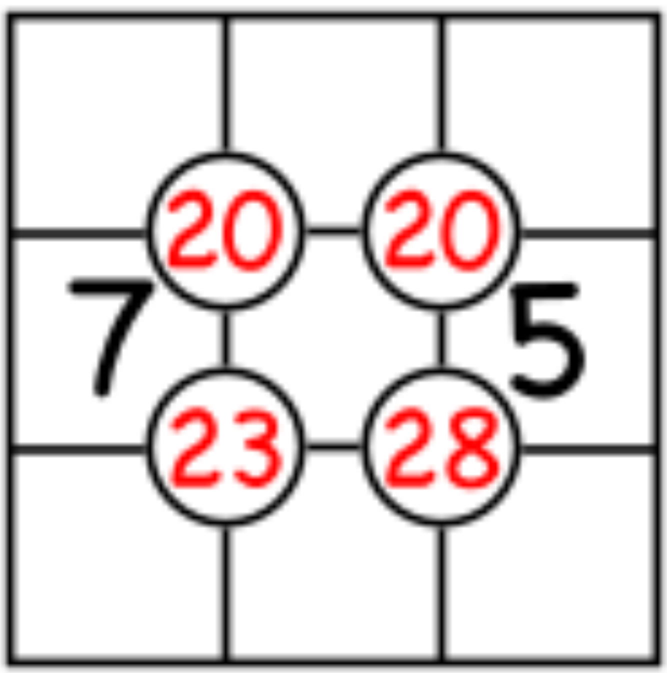
3+7+4+9 = 23

4+9+8+5 = 26

7+1+9+2 = 19

9+2+5+6 = 22

THE CHALLENGE: Fill in this Sujiko puzzle.



1 2 3 4 6 8 9

Puzzle of the Week
Sujiko Puzzle – 3 – Notes

THE CHALLENGE: When solving a puzzle, look for its weak points, its easiest points of entry.

For this puzzle, the 28 in the circle is quite a big number and seems a good place to start. The three missing numbers in the bottom right 2 by 2 corner must add up to 28. The only way to get 28 from the list of numbers we have is to use 6, 8, and 9. So, we know those three numbers must be in that bottom right corner (and nowhere else).

Looking at the upper right 2 by 2 corner, 3 and 4 are the largest remaining numbers, so the central square must be 8 or 9. This means that the two numbers on the right side of the upper row are (3 4) when 8 is in the middle, or (2 4) when 9 is in the middle.

Suppose the central square is 8. This means the right side of the upper row is (3 4).

> Looking at the bottom left 2 by 2 corner, the two squares shared with the bottom right 2 by 2 corner are (8 6) or (8 9). However, (8 9) makes it impossible to get a sum of 23 in the bottom left 2 by 2 corner. Therefore, it must be (8 6), and so the bottom left corner must be 2.
>
> The only unused value is 1, which must go in the upper left corner. That makes the sum of the upper left corner either 1 + 3 + 7 + 8 = 19 or 1 + 4 + 7 + 8 = 20. The latter of the two is a solution!

Suppose the central square is 9. This means the right side of the upper row is (2 4).

> Looking at the bottom left 2 by 2 corner, the two squares shared with the bottom right 2 by 2 corner are (9 6) or (9 8). However, (9 8) makes it impossible to get a sum of 23 in the bottom left 2 by 2 corner. Therefore, it must be (8 6), and so the bottom left corner must be 1.
>
> The only unused value is 3, which must go in the upper left corner. That makes the sum of the upper left corner either 3 + 2 + 7 + 9 = 21 or 3 + 4 + 7 + 9 = 23, neither of which work!

Therefore, the only solution is given by (showing triplets as rows):

(1 4 3)

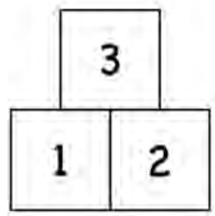
(7 8 5)

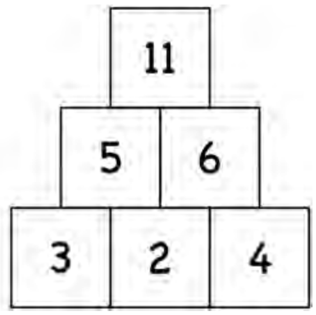
(2 6 9)

Puzzle of the Week

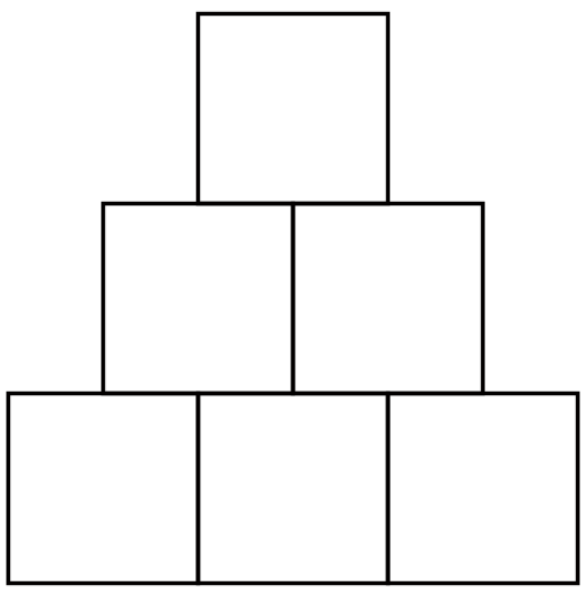
Sum Pyramids – 1

These pyramids are called *Sum Pyramids*. The number above each pair of connected numbers is their sum.





THE CHALLENGE: Place some of the numbers from 1 to 10, not repeating any number, to make a Sum Pyramid with the smallest possible number on top. Can you do better than 11?



1 2 3 4 5 6 7 8 9 10

Puzzle of the Week

Sum Pyramids – 1 – Notes

THE CHALLENGE: The center square in the bottom right will contribute to both squares in the middle row, and so its value will show up twice in the top number. To keep the top number as small as possible, we should put a 1 in the middle of the bottom row. We would like to put 2 and 3 in the two remaining squares in the bottom row, but that is not possible because 1 + 2 = 3 would cause 3 to be used twice.

To make the sum as small as possible, we will use (2 1 4) in the bottom row. This produces the pyramid:

(8)

(3 5)

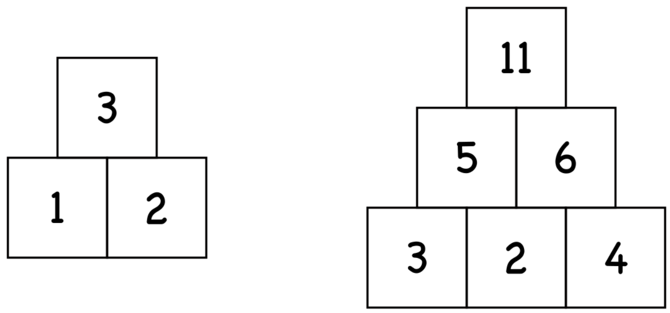
(2 1 4)

This is a considerable improvement over having 11 on top!

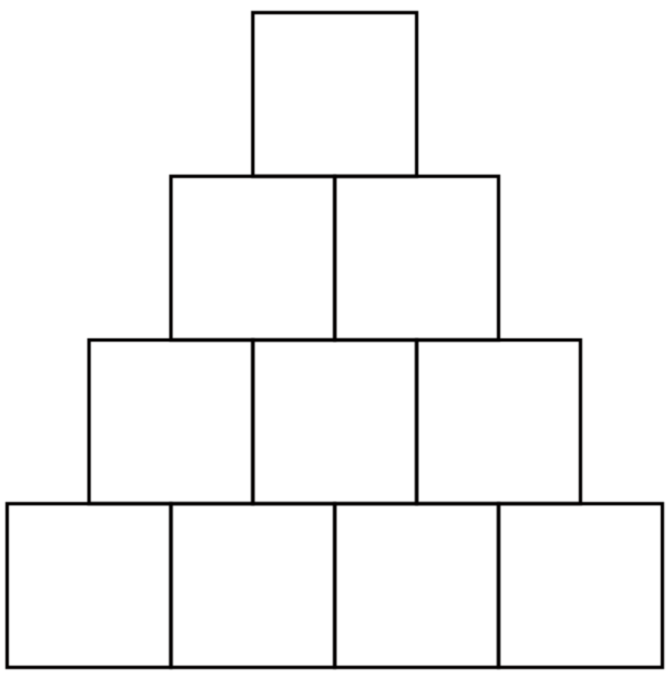
Puzzle of the Week

Sum Pyramids – 2

These pyramids are called *Sum Pyramids*. The number above each pair of connected numbers is their sum.



THE CHALLENGE: Place some of the numbers from 1 to 25, not repeating any number, to make a Sum Pyramid with the smallest possible number on top. Can you do better than 25?



Puzzle of the Week

Sum Pyramids – 2 – Notes

THE CHALLENGE: The pyramid is completely determined by the entries in the bottom row.

Each of the middle numbers in the bottom row will contribute a total of three times to the number on the top of the pyramid. The corner numbers in the bottom row will each only contribute once to the number on the top of the pyramid. Therefore, let's use 1 and 2 for those middle numbers on the bottom row.

1 and 2 creates a 3 on the next row, so we can't use it in the bottom row. Exploring how to finish out the bottom row we should consider (4 1 2 5), (5 1 2 4), (4 1 2 6), and (6 1 2 4). All of these produce duplicates. Looking further, we can try (5 1 2 7) or (6 1 2 8). However, (5 1 2 7) produces a duplicate entry, and (6 1 2 8) has a top number of 23, which is only a little better than 25.

Going back, let's consider using 1 and 3 in the middle of the bottom row. Some exploration produces the following, which is our best answer.

(20)

(11 9)

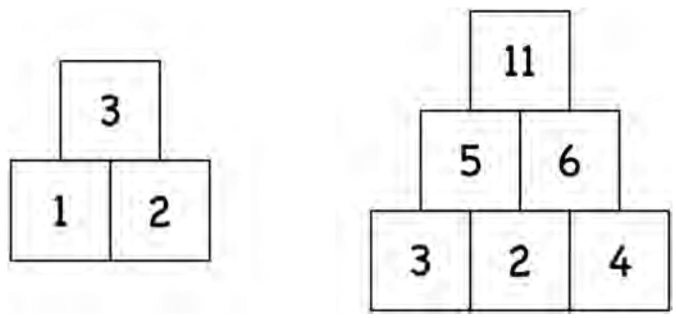
(7 4 5)

(6 1 3 2)

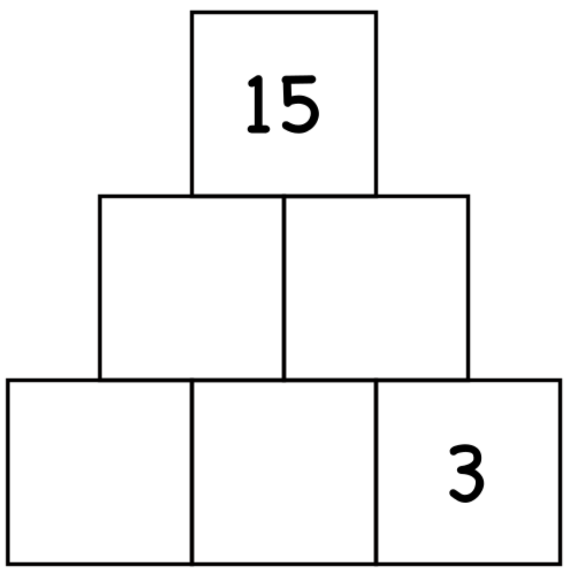
Puzzle of the Week

Sum Pyramids – 3

These pyramids are called *Sum Pyramids*. The number above each pair of connected numbers is their sum.



THE CHALLENGE: Place some of the numbers from 1 to 15, not repeating any number, to complete this Sum Pyramid. Can you find more than one solution?



Puzzle of the Week

Sum Pyramids – 3 – Notes

THE CHALLENGE: Following the numbers up, 15 will equal 3 plus twice the middle number in the bottom row plus the leftmost number in the bottom row. So, 12 will be twice the middle number plus the leftmost number.

The possibilities for the bottom row are: (2 5 3), (4 4 3), (6 3 3), (8 2 3), and (10 1 3). This gives us three possible solutions:

(15)

(7 8)

(2 5 3)

or

(15)

(10 5)

(8 2 3)

or

(15)

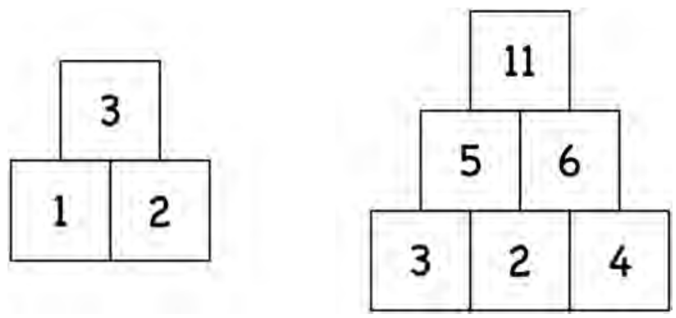
(11 4)

(10 1 3)

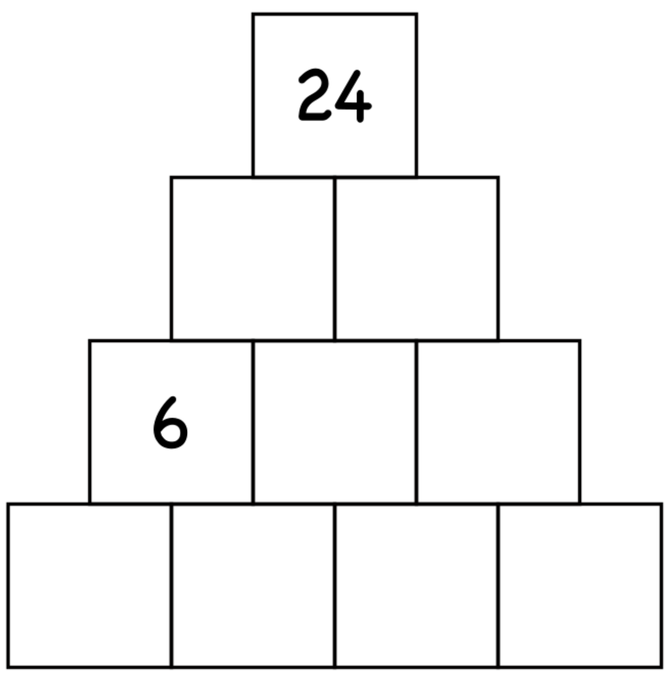
Puzzle of the Week

Sum Pyramids – 4

These pyramids are called *Sum Pyramids*. The number above each pair of connected numbers is their sum.



THE CHALLENGE: Place some of the numbers from 1 to 24, not repeating any number, to complete this Sum Pyramid. Can you find more than one solution?



Puzzle of the Week

Sum Pyramids – 4 – Notes

THE CHALLENGE: The puzzle is completely determined by its bottom row.

Tracing numbers up, we have two immediate constraints on the bottom row. The leftmost two numbers on the bottom row must add up to 6, so they are 1 + 5 or 2 + 4. Also, three times the sum of the two middle numbers, plus the sum of the two corner numbers, must be 24.

There are four possible contributions from the left side of the bottom row. In each case, we see the right side will contribute the remainder using 3 times its central number plus 1 times its corner number.

Case 1: (1 5) which contributes 1 + 3 x 5 = 16. We need 8 more from the right side. That cannot be done without duplicating numbers.

Case 2: (5 1) which contributes 5 + 3 x 1 = 8. We need 16 more from the right side. This can work with (3 7) or (2 10). If the bottom is (5 1 3 7), there is a duplication. If the bottom is (5 1 2 10), it works!

Case 3: (2 4) which contributes 2 + 3 x 4 = 14. We need 10 more from the right side. This can work with (3 1) or (1 7). (f the bottom is (2 4 3 1), there is a duplication. If the bottom is (2 4 1 7), it works!

Case 4: (4 2) which contributes 4 + 3 x 2 = 10. We need 14 more from the right side. This can work with (3 5) or (1 11). If the bottom is (4 2 3 5), there is a duplication. If the bottom is (4 2 1 11), it works!

Putting this together, there are the three solutions:

(24)
(9 15)
(6 3 12)
(5 1 2 10)

or

(24)
(11 13)
(6 5 8)
(2 4 1 7)

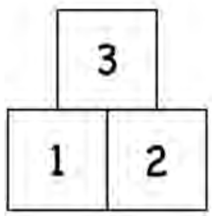
or

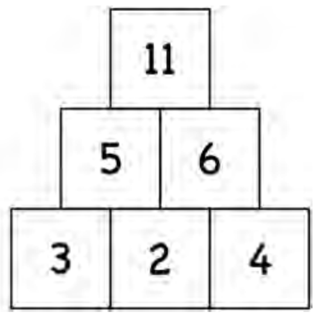
(24)
(9 15)
(6 3 12)
(4 2 1 11)

Puzzle of the Week

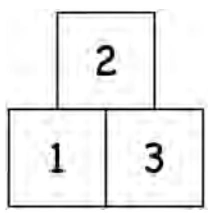
Sum Pyramids – 5

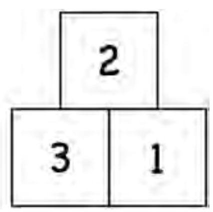
These pyramids are called *Sum Pyramids*. The number on top is the sum of the two numbers below it.





These pyramids are called *Difference Pyramids*. The number on top is the difference of the two numbers below.





THE CHALLENGE: Describe how any Sum Pyramid can easily be turned into a Difference Pyramid. Can you find an example of a Difference Pyramid that cannot be easily turned into a Sum Pyramid?

Puzzle of the Week

Sum Pyramids – 5 – Notes

THE CHALLENGE: To make a Difference Pyramid out of any Sum Pyramid, simply rotate the Sum Pyramid onto its left or right side!

What can go wrong in wanting to turn a Difference Pyramid into a Sum Pyramid, is that the differences may not always go in the same direction. Neither of the following Difference Pyramids can be turned into a Sum Pyramid without a complete rearrangement of the numbers. That's because they both have a 6 in the middle of a side, and any Sum Pyramid with the numbers 1 to 6 must have the 6 at the tip.